Beam Bending and Beam Stiffness

Consider a uniform beam subjected to a ***bending moment***, M, as shown in Fig.12.1. The axial strain, ε, at a distance y from the ***neutral axis*** is given by

$$\varepsilon = \frac{y}{R} = \kappa y \qquad (12.1)$$

where R is the radius of curvature of the beam and the reciprocal of this, κ, is termed its ***curvature***. The axial stress at any point can therefore be written as the product of the Young's modulus, E, the distance from the neutral axis, y, and the beam curvature, κ

$$\sigma = E\varepsilon = E\kappa y \qquad (12.2)$$

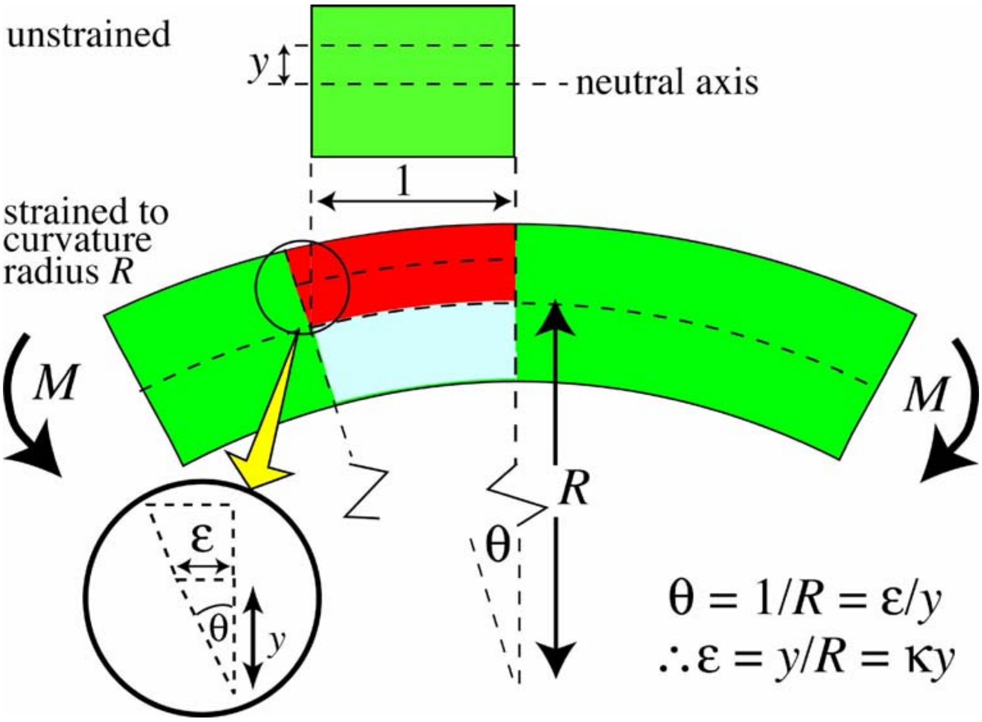

Fig.12.1 Strains induced during bending of a beam by the application of a moment M

Bending moments are commonly produced by transverse loads of some sort. Consider the cantilever beam shown in Fig.12.2. The moment acting on a section of the beam is the product of the applied force and its distance from the section. It is balanced by the internal moment arising from the stresses generated. This is given by

$$M = \int y(\sigma \mathrm{d}A) = \int y(E\kappa y\,\mathrm{d}A) = \kappa E\int y^2 \mathrm{d}A = \kappa EI \qquad (12.3)$$

where dA is an element of the section at a uniform distance, y, from the neutral axis and I, the ***second moment of area*** ("***moment of inertia***") is dependent on the sectional shape.

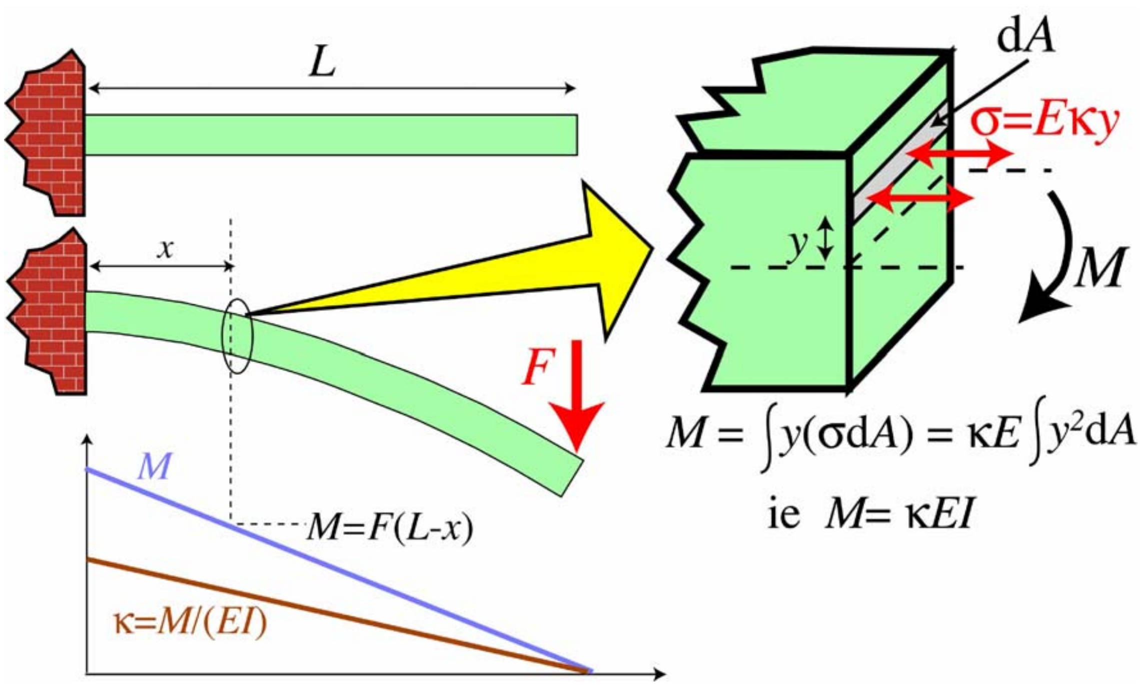

Fig.12.2 Balancing of external and internal moments during beam bending

The product *EI* is termed the "***beam stiffness***", or sometimes the "***flexural rigidity***". It's often given the symbol Σ. It's a measure of how strongly the beam resists deflection. The beam curvature, κ, is thus given by the ratio of the applied moment to the beam stiffness, in an analogous way to the axial strain being equal to the ratio of applied stress to Young's modulus, under uni-axial loading.