Brittle Fracture and Derivation of G_c

Crack-initiated Fracture

Systematic work on the effect of prior cracks was first carried out by A.A. Griffith in the 1920's. He combined the ***fracture energy*** concept with the expected effect of the presence of a crack. Consider the ellipsoidal crack shown in Fig.9.2. Inglis (1913) had established that stress is concentrated at the tip of such a crack, such that the peak stress is

$$\sigma_{max} = \sigma_0 \left[1 + 2\left(\frac{c}{r}\right)^{1/2} \right] \tag{9.5}$$

where $2c$ is the crack length (c for a surface crack) and r is the radius of curvature at the tip. (It follows that the stress concentration factor for a circular hole is 3.)

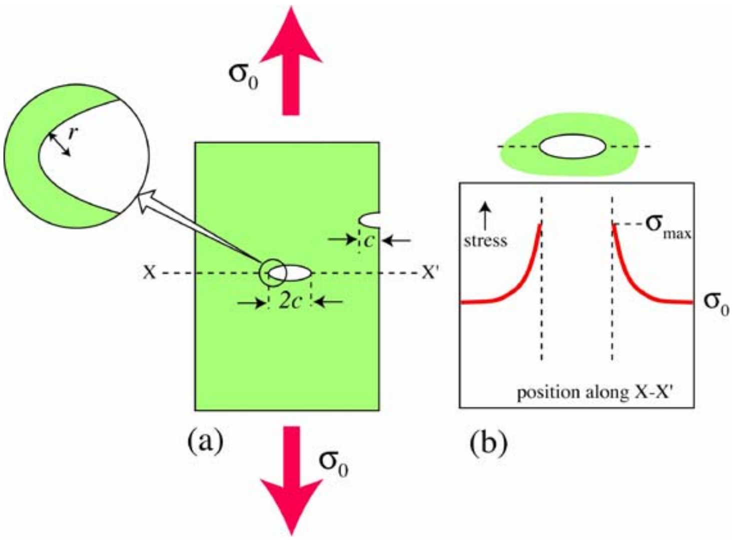

Fig.9.2 Stress concentration at a crack tip: (a) loading of a flat plate containing an ellipsoidal crack and (b) schematic stress distribution in the vicinity of the crack.

One approach to predicting whether the crack will propagate is to equate this peak stress to the theoretical strength, given by eqn.(9.4). However, this is not very satisfactory. Firstly, the crack tip radius may be difficult to establish. More fundamentally, it gives a condition necessary for fracture, but not sufficient. Not only must the stresses at the crack tip be sufficient to break bonds, but also the process must be energetically favourable.

Energetics of Crack Propagation

The driving force for crack advance comes from elastic strain energy stored in the stressed material. As a crack gets longer, the volume of stress-free material "shielded" by it from the applied stress increases (Fig.9.3). The strain energy stored per unit volume in (elastically) stressed material is

$$U = \frac{1}{2} \sigma \varepsilon = \frac{\sigma^2}{2E} \tag{9.6}$$

so the energy released when the crack extends (at both ends) by dc is the product of this expression and the increase in stress-free volume. The shape of the stress-free region is not well-defined, and the stress was not uniform within it before crack advance anyway, but taking the relieved area to be twice that of the circle having the crack as diameter gives a fair approximation. Thus, for a plate of thickness t, the energy released during incremental crack advance is given by

$$\mathrm{d}W = \frac{\sigma_0^2}{2E} 2(2\pi c \ t \ \mathrm{d}c) = \frac{2\sigma_0^2 \pi c \ t \ \mathrm{d}c}{E} \tag{9.7}$$

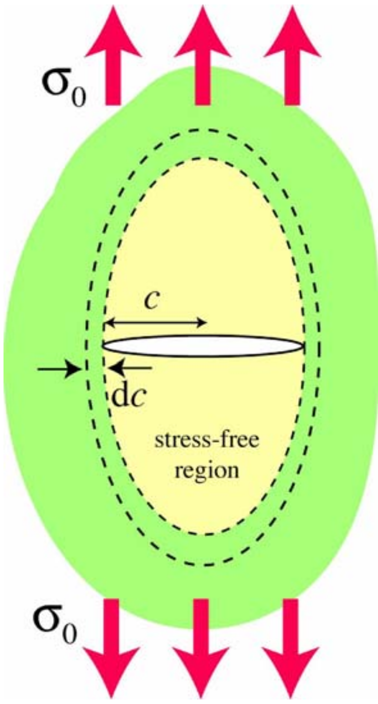

Fig.9.3 Stress-free region shielded by a crack from the applied load

The Strain Energy Release Rate

The concept of stored elastic strain energy being released as the crack advances is central to fracture mechanics. The ***strain energy release rate*** (crack driving force) is usually given the symbol G (not to be confused with shear modulus or Gibbs free energy!). It is a "rate" with respect to the creation of new crack area (so has units of J m^{-2}) and does not relate to time in any way. It follows that

$$G = \frac{\mathrm{d}W}{\text{new crack area}} = \frac{\left\{\left(2\sigma_0^2 \pi c t \ \mathrm{d}c\right)/E\right\}}{(2 \ t \ \mathrm{d}c)} = \pi\left(\frac{\sigma_0^2 c}{E}\right) \qquad (9.8)$$

The value of the constant (π in this case) is not well defined. It depends on specimen geometry, crack shape/orientation and loading conditions. In any event, the approximation used for the stress-free volume is simplistic. However, the dependence of G on ($\sigma_0^2 c/E$) is more general and has important consequences.

The Fracture Energy (Crack Resistance)

In order for crack propagation to be possible, the strain energy release rate must be greater than or equal to the rate of energy absorption (expressed as energy per unit area of crack). This energy requirement is sometimes known as the ***Griffith criterion***. Focussing now on an edge (surface) crack of length c, propagating inwards, for a brittle material this ***fracture energy*** is simply given by 2 γ (where γ is the ***surface energy***, with the factor of 2 arising because there are two new surfaces created when a crack forms). It can be considered as a ***critical strain energy release rate***, G_c. It is a material property. It is sometimes termed the ***crack resistance***. The ***fracture strength*** can thus be written as

$$G \ \geq \ G_c = 2\gamma$$

$$\therefore \pi\left(\frac{\sigma_0^2 c}{E}\right) \ \geq \ 2\gamma \qquad (9.9)$$

$$\therefore \sigma_* = \left(\frac{2\gamma E}{\pi c}\right)^{1/2}$$