Redefining the Concept of Stress-Strain Curve for Foams.

Tomasz Wierzbicki[1], Mulalo Doyoyo[1] and Athina Markaki[2]

[1]Impact and Crashworthiness Laboratory
Massachusetts Institute of Technology
Cambridge, MA, USA

[2]Department of Materials Science and Metallurgy
Cambrigde University
Cambridge, UK

Abstract. The goal of this paper is to develop a new procedure for determining the actual strains in uniaxially-compressed aluminum foams and obtain the stress-strain curves. The drawback of using cubic tests is that only average strains can be calculated. This method combined with an intrinsic inhomogeneity of the commercial foam introduces a large size effect. The new procedure removes this deficiency and can be applied efficiently in finite element analysis. Compressive tests were performed on tapered blocks of Alporas foam of different semi-wedge angles (75^o and 80^o). A crushing front propagated into the specimen with loading. The deformed specimen develops a new shape which allows for the calculation of actual strains. It is thus recommended that tapered specimens be used to determine the compressive properties of foam. Here, the stress-strain curve of foams is described by three parameters; the initial plateau stress, the shape exponent and the densification strain, all obtained from a single test.

1 Introduction

This paper deals with the determination of stress-strain curve of closed-cell foams for FE applications. In FE models, the size of a solid element and the constitutive equation are specified. In continuum mechanics, the constitutive relation is independent of the spatial discretization. For foams, the size of the solid element is related to its mechanical properties. Bastawros and Evans [1] detected bands of concentrated strain spaced 3 to 4 cell distance apart. The strain levels in the cells between these bands were much lower. The actual strains in the foam cubes are thus different from the average strains obtained by dividing the displacement by the cube height. In problems involving strain gradients, such as indentation of a punch into foam, a distinct boundary separates crushed cells from "undeformed" cells (Fig. 1).

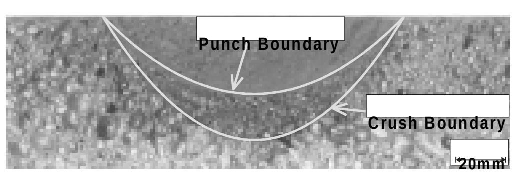

Figure 1: Distinct crushing boundary in hemispherical indentation of Al foam [2].

The crushing stress is the indentation force divided by the current punch cross-sectional area (Doyoyo and Wierzbicki, [2]), but the strain is difficult to measure. FE codes require the actual stress-strain curve for the material, which can not be obtained from indentation nor from uniaxial cubic tests. A new test which involves crushing of tapered specimens was developed. From the deformed shape, actual stresses and strains can be calculated. The use of tapered specimens to measure material properties is not new. Marciniak [3] measured the stress-strain curve from copper cones. McClintock and Zheng [4] used tapered specimens to study fracture in ductile metals. The present technique is a promising way of acquiring local crush properties of foams which can be used in FE modeling.

2 Analysis of Non-Uniform Crushing of a Foam

Consider the tapered specimen in Fig. 2. As force P increases, a front which separates crushed and undeformed cells is propagated. For $\nu = 0$, the point A moves down vertically to the new position A'. The true stress is

$$\sigma = \frac{P}{w_o\,(b - 2x)} \tag{1}$$

where $b = b\,(P)$. The yield stress σ_o should remain constant at the front

$$\sigma_o = \frac{P_o}{w_o b_o} = \frac{P}{w_o b} \tag{2}$$

where P_o is the initial peak load. Alternatively,

$$\sigma = P\left\{w_o\left(b_o\frac{P}{P_o} - 2x\right)\right\}^{-1} \tag{3}$$

which is more convenient because P is readily measurable, and b is not.

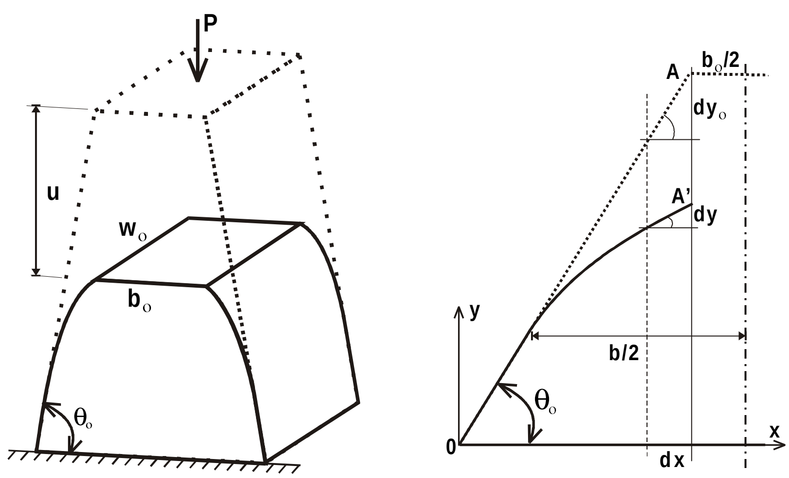

Figure 2: Geometry of the deformed and undeformed tapered specimen.

The engineering and true strains are

$$E = \frac{dy_o - dy}{dy_o} = 1 - \frac{y'}{(y')_o} \quad \text{and} \quad \varepsilon = \ln\frac{dy}{dy_o} = \ln\frac{y'}{(y')_o} \tag{4}$$

where $(y')_o = dy_o/dx = \tan\theta_o$ and $y'(x) = dy(x)/dx$

3 Densification and Locking

Gibson and Ashby [5] defined densification strain as

$$E_d = 1 - 1.4\frac{\rho_f}{\rho_{Al}} \tag{5}$$

E_d represents the onset of densification, which is not clear because hardening of foam proceeds gradually and E_d is not unique. Hanssen *et al.* [6] defined E_d as

$$E_d = 1 - \frac{\rho_f}{\rho_{Al}} \tag{6}$$

E_d represents the complete closure of all cells. For $\nu = 0$, the above expression for E_d can be derived from the *work conjugancy* relation

$$\frac{1}{\rho}\sigma d\varepsilon = \frac{1}{\rho_f}SdE \tag{7}$$

noting that $\rho = \rho_{Al}$ when $E = E_d$, and using the *strain conjugancy* relation between ε and E, Eq. 6 is recovered. The plastic Poisson's ratio ν is

$$\nu = -\frac{E_{yy}}{E_{xx}} \tag{8}$$

but varies with E from ν_o to $\nu_L = 0.5$ at densification. The value $\nu_o \approx 0$ for low density foams. The variation of ν with E has not been reported in the literature, and $\nu(E)$ is assumed to follow the power law

$$\nu = \nu_o + \beta E^m \tag{9}$$

At densification,

$$0.5 = \nu_o + \beta E_d^m \tag{10}$$

The accumulated Poisson's ratio is measured at full densification

$$\nu_{acc} = -\frac{(E_{yy})_d}{(E_{xx})_d} = \int_0^{E_d} (\nu_o + \beta E^m)\, dE \tag{11}$$

From Eqs. (10) and (11), the two unknown parameters are

$$m = E_d\frac{0.5 - \nu_o}{\nu_{acc} - \nu_o} - 1 \quad \text{and} \quad \beta = \frac{0.5 - \nu_o}{E_d^m} \tag{12}$$

4 Tests with Tapered Specimens

The tapered specimens are made of Alporas foam (Shinko Wire Company, Amagasaki, Japan). Its cell walls are made of Al with small percentages of Ca and Ti. The nominal relative density $\rho* = 9.5$ % and the average cell size is 3.7mm. The two tapered specimens had $\theta_o = 75^o$ and 80^o, $h_o = 300$mm and $w_o = 100$mm, with $b_o =$ 75 mm and 100 mm for 75^o and 80^o tapered blocks respectively. The tests were performed on an MTS testing machine (Model 45G, MTS, Eden Prairie, MN) with a 200 kN load cell . The specimens were placed between two parallel platens and

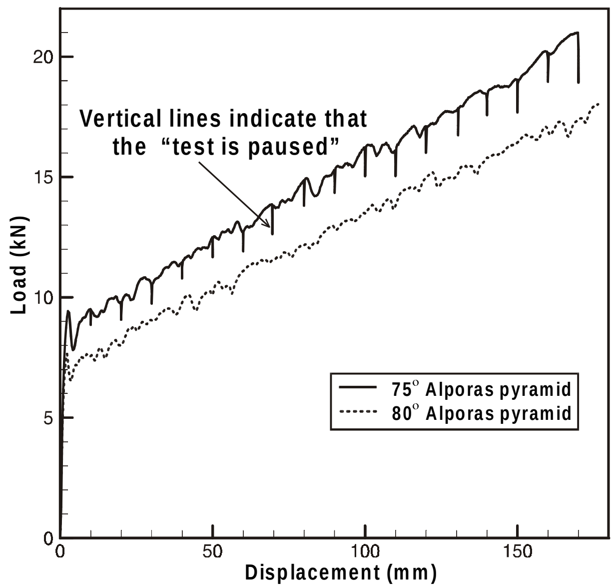

Figure 3: Load-displacement response for 75^o and 80^o tapered blocks.

then compressed. Force and displacement were acquired using TestWorks software (Sintech Division, MTS). The 75^o tapered block is compressed at a loading rate of 0.05mm/sec to a height of 130mm. The 80^o tapered block is compressed at a loading rate of 0.10mm/s to a height of 125mm. The load-displacement curve for the tests are shown in Fig. 3. The 75^o tapered block, before and after the test is shown in Fig. 4. Figure 5 shows the final shapes of the two blocks. In Fig. 3, the load increases linearly with displacement. If best fit lines are drawn through the data points, the loads P_o at which the lines intersect the load axis is called the *initial peak load*. Thus, $\sigma_o = 0.91$ MPa and $\sigma_o = 0.93$ MPa for the 75^o and 80^o tapered blocks respectively.

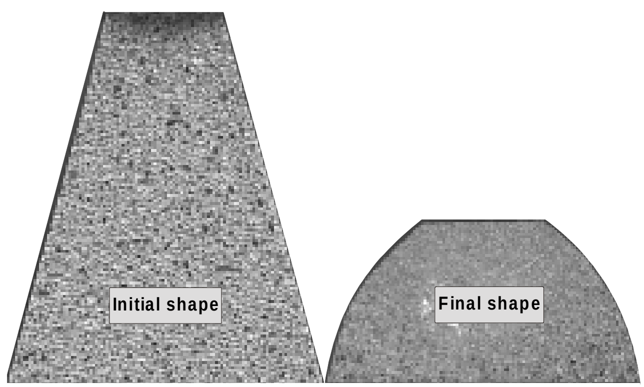

Figure 4: The 75^o Alporas tapered block, before and after testing.

The data (x_i, y_i, i = 1,2,3,...,N) of the digitized image of the deformed specimens are linear in $\ln(\tan\theta_o x_i - y_i)$ vs $\ln x_i$ space. Figure 6 shows how the deformed shape was determined from the digitized points, while Fig. 7 shows the fitted functions. Thus, a two parameter representation of the deformed edge of the block gives very accurate results

$$y = \tan\theta_o x - \alpha x^n \tag{13}$$

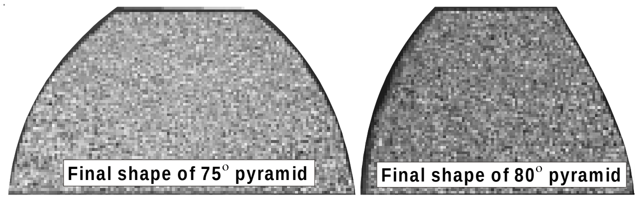

Figure 5: Final shapes of 75^o and 80^o Alporas tapered blocks after testing.

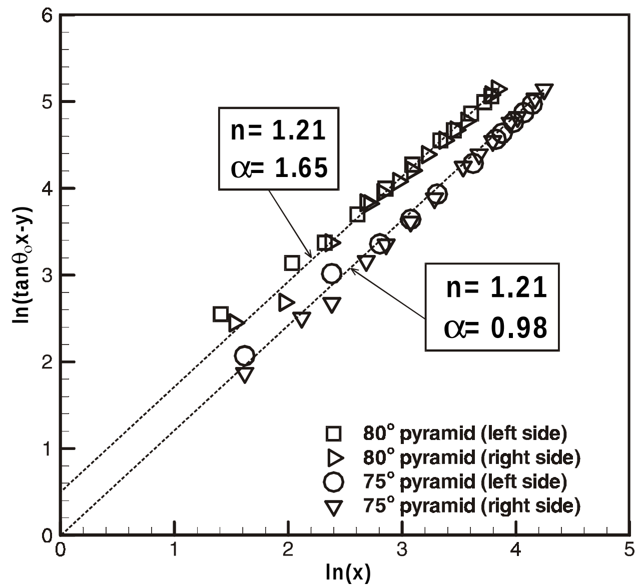

Figure 6: Evaluation of n and α for the 75^o and 80^o tapered blocks.

The engineering strain is

$$E = \frac{\alpha}{\tan\theta_o} n x^{n-1} \tag{14}$$

Eliminating x from Eq. (1) using the above expression, the stress becomes

$$\sigma = S = P\left\{w_o\left(b_o\frac{P}{P_o} - 2\left(\frac{\tan\theta_o E}{\alpha n}\right)^{\frac{1}{n-1}}\right)\right\}^{-1} \tag{15}$$

Equation (15) has a vertical asymptote when the strain reaches E_d

$$b_o\frac{P}{P_o} - 2\left(\frac{\tan\theta_o E_d}{\alpha n}\right)^{\frac{1}{n-1}} = 0 \tag{16}$$

Combining Eq. (15) with (16), one gets

$$\frac{\sigma}{\sigma_o} = \left\{1 - \left(\frac{E}{E_d}\right)^{\frac{1}{n-1}}\right\}^{-1} \tag{17}$$

Transforming E to the ε, using the convention $(E, \varepsilon) \geq 0$

$$\varepsilon = -\ln(1 - E) \tag{18}$$

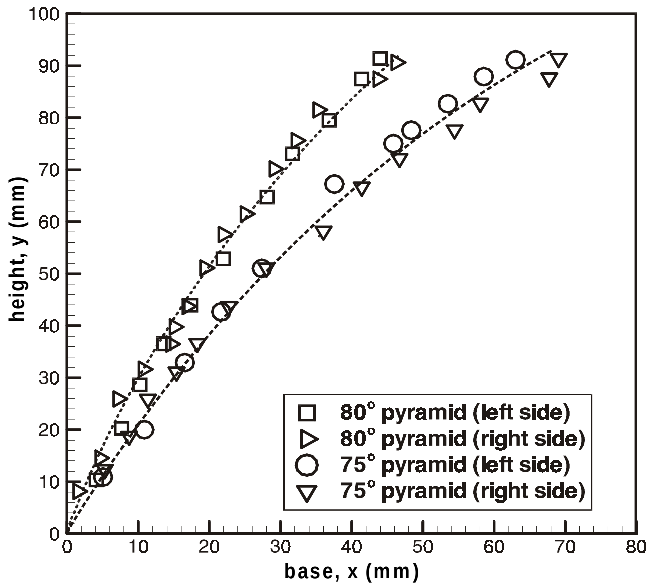

Figure 7: Analytical fit of the final deformed shape vs. digitized data points.

the true stress-strain curve can then be derived from Eq. (17) as

$$\frac{\sigma}{\sigma_o} = \left[1 - \left(\frac{1 - exp\{-\varepsilon\}}{1 - exp\{-\varepsilon_d\}}\right)^{\frac{1}{n-1}}\right]^{-1} \tag{19}$$

Plots of Eq. (17) and (19) for $\sigma_o = 1$ MPa, and $E_d = 0.9$ are compared in Fig. 10 for different n. For 75^o specimen; $n = 1.21$, $\alpha = 0.98$, $b = 240$mm and $E_d = 0.87$, while for 80^o specimen, $n = 1.21$, $\alpha = 1.65$, $b = 163$mm and $E_d = 0.89$.

A block of Alporas foam with $h_o = 27$mm, $w_o = 50$mm and $b_o = 50$mm is compressed in the MTS machine at 0.05mm/s until $P \approx 80P_o$. Figure 8 gives the stress-strain curve, with pictures of the crushed and undeformed block. $E_d = 0.91$ which is equal to E_d calculated from Eq. (6) with $\rho_f = 9.5$ %. The crushed cross-section is 54.5mm by 54.mm. Thus the lateral strain at full densification $(E_{yy})_d = 0.09$, so that for $E_d = 0.9$, $\nu_{acc} = 0.09/0.9{=}0.1$; hence $m = 3.5$ and $\beta = 0.72$, and $\nu(E)$ is shown in Fig. 9. The stress-strain curves are compared for the tapered and cubic ($h_o = 17$, 42, and 84mm, $A_o = 50$mm x 50mm) specimens in Fig. 11. The shift is observed because of a large difference between the average strain in cubic tests and the actual strain in tapered tests.

5 Conclusion

A procedure to determine actual stress-strain curves for foams has been presented. The strains and stresses are calculated from the deformed shape of a tapered block after compression. Tests were conducted on Al foam blocks of taper angles: 75^o, 80^o. A boundary separated the crushed cells from the undeformed cells during loading. A two-parameter function described the deformed shapes of the specimens. The stress-strain curve was described by three parameters; initial plateau stress, shape exponent and densification strain, all obtained from a single test. In the future, scaling relations between average and actual strains will be determined. The effect of the variable Poisson's ratio on the stress-strain curve will also be investigated.

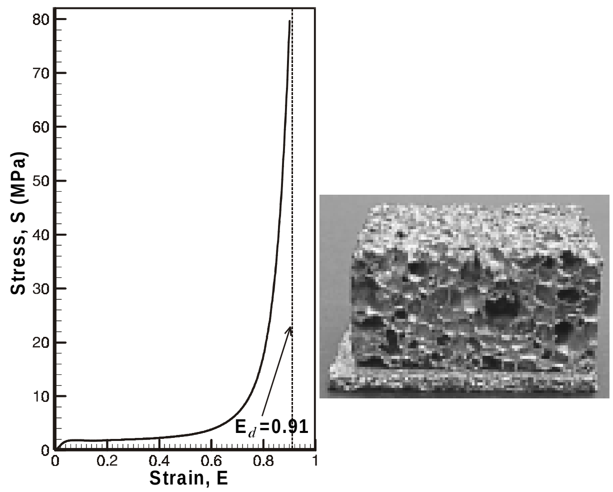

Figure 8: Complete foam crushing with pictures of crushed and original block.

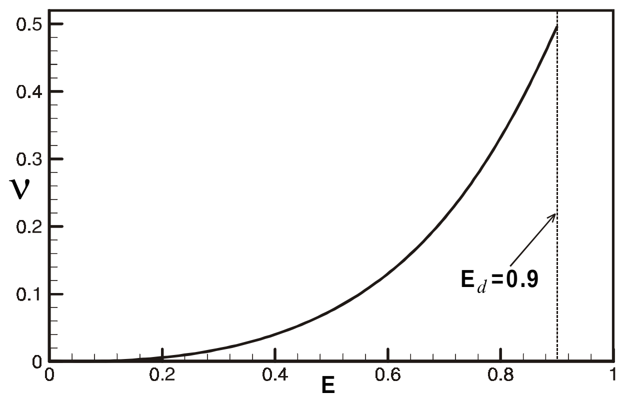

Figure 9: ν as a function of E for the Alporas foam.

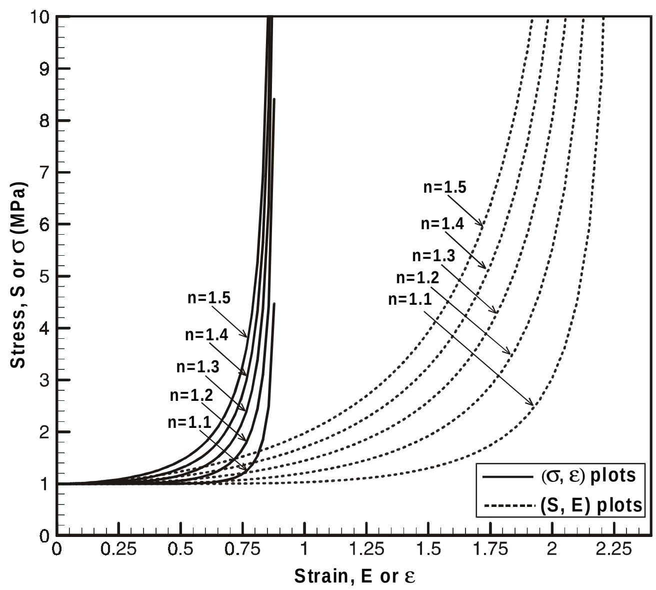

Figure 10: Stress vs strain responses for different values of n, according to Eq. (17).

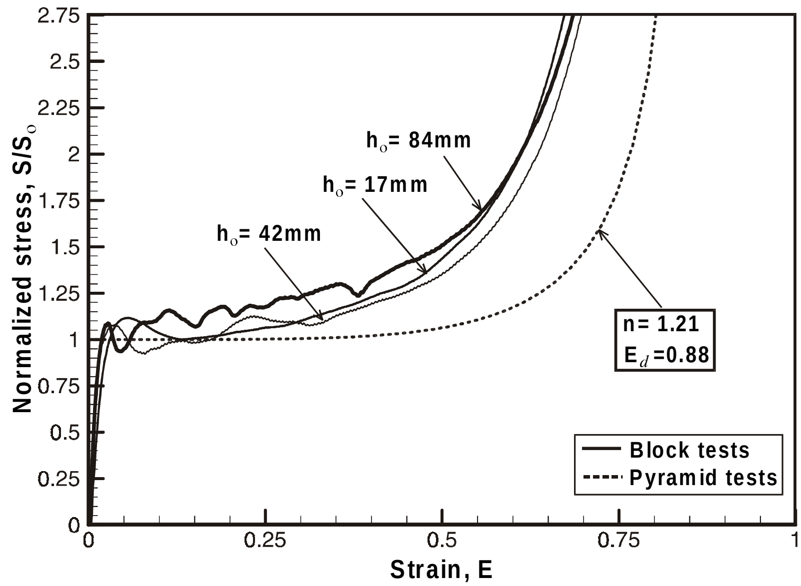

Figure 11: Comparison of stress-strain curves between tapered and rectangular block tests.

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