The behaviour of *magnesium* crystals is shown in Fig. 108. The mean ultimate glide strain decreases clearly with increasing alloy content, a phenomenon due to the growing tendency to glide fracture along the basal plane. In order to assess technologically the effect of alloying, one has to bear in mind not only the increase in the critical shear stress of the pure metal ($v_{\text{all.}}$) but also the subsequent capacity for work hardening. Accordingly the total hardening which can be achieved by alloying *and* glide to the point of fracture $\left(v_{\text{all.}} \cdot v_{\text{Tr}} = \dfrac{S_{0,\,\text{all}}}{S_0} \cdot \dfrac{S_{e,\,\text{all.}}}{S_{0,\,\text{all.}}} = \dfrac{S_{e,\,\text{all.}}}{S_0}\right)$ is plotted in Fig. 109 as a function of the concentration.

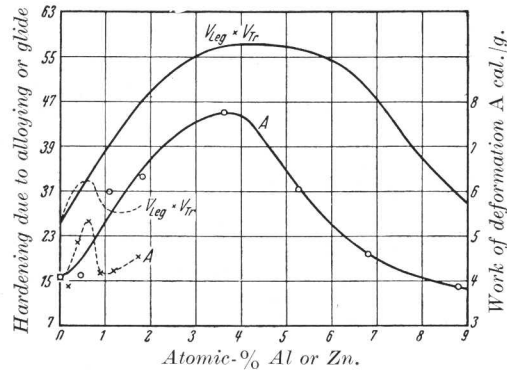In both cases it will be found that it is not the maximum but a

FIG. 109.—Work-hardening Capacity of Mg Solid Solutions (231).
———— Al–Mg, – – – – Zn–Mg.

medium concentration to which the maximum value of this total hardening corresponds. The cause of this must no doubt be sought in the increase of brittleness, already referred to, which accompanies an increase in alloying content. The work of deformation behaves in a similar way to that of the product $v_{\text{all.}} \cdot v_{\text{Tr}}$.

The relationship between the yield–stress curves and the concentration of cubic face-centred *zinc–aluminium* solid solutions is shown in Fig. 110. The hardening caused by zinc additions, particularly noticeable at the start of extension, is apparent from these curves. The lattice rotation which accompanies stretching of the crystal corresponds to the normal case of octahedral glide which was described in Section 36. At higher zinc contents the symmetrical position of the crystal axis, which leads to double glide,

is not attained, because the crystals had fractured within the region of simple glide without necking. This fracture appears to be a sort of shear fracture on the operative glide plane, similar to that observed with magnesium solid solutions.

The yield–stress curves of α-brass crystals of various zinc concentrations are shown in Fig. 111. In spite of the initially increased yield stress, the shear stress in the operative octahedral system remains during extension below that of the pure copper crystal. The higher the zinc content the lower is this shear stress. The lattice rotation which accompanies glide again corresponds essen-
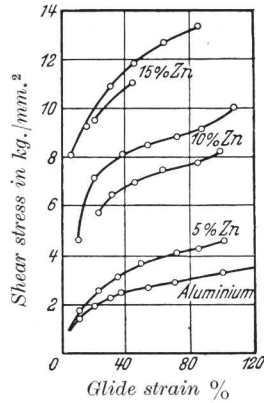

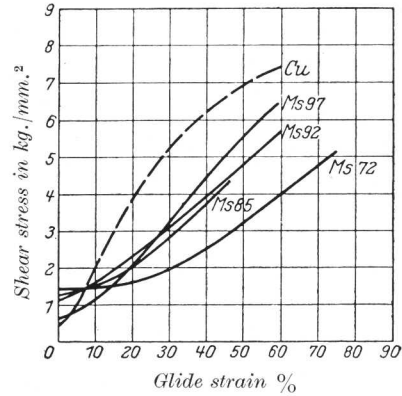FIG. 110.—Yield–Stress Curves of Zn–Al Solid Solutions (239).

FIG. 111.—Yield–Stress Curves of α-Brass Crystals (234).

tially to the typical case for octahedral glide. The differences relate only to the interaction between the two operative glide systems. The higher the zinc content, the more will the exclusive operation of the first system extend beyond the symmetry position (Fig. 112). This means that the more zinc there is in the crystals the more will the hardening of the initially latent octahedral system exceed that of the operative system. Fig. 113 gives an example for the variation of the critical shear stress in the two glide systems. The second system becomes active when its resolved shear stress exceeds that of the first system by 30 per cent. In the course of further extension the symmetry position (equality of both shear stresses) is again exceeded, this time in the opposite direction because it is now the first system, temporarily latent, which experiences the greater hardening. Only under the influence of a

shear stress that is 10 per cent. higher than that of the operative system will it again become active. The higher the shear stress at the start of glide, the less marked will be the subsequent hardening with increasing extension. The new bands which appear with each change in the glide system are shown in Fig. 114. The ultimate tensile stress of crystals with 72 per cent. copper is between 12·9 and 32·9 kg./mm.², the elongation between 67 and approx. 122 per cent.—according to their initial orientation (233).
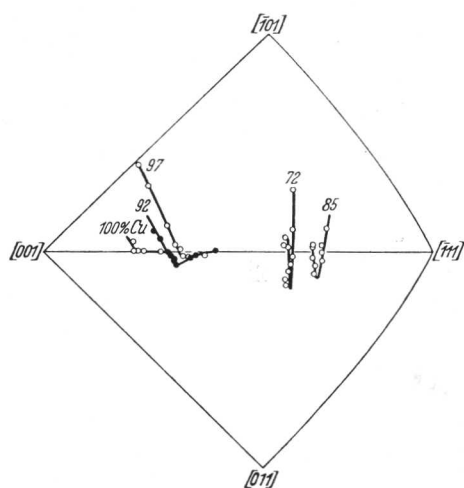
Fig. 112.—Unequal Rate of Hardening of the Operative and Latent Octahedral System in the Extension of α-Brass Crystals, Expressed in Relation to Concentration.
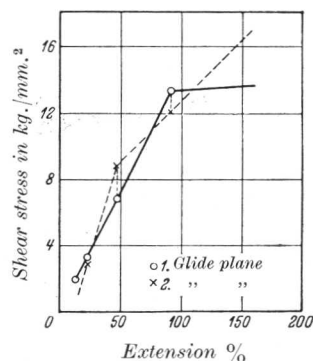
Fig. 113.—Variation of the Critical Shear Stress in the Double Glide of α-Brass Crystals with 70% Cu (240); cf. also (233).

Copper–aluminium solid solutions exhibit a behaviour substantially analogous to that of the α-brass crystals (241). In this case, too, if the initial shear stress is high the yield–stress curve soon intersects that of the pure copper crystal, and exhibits for the higher glide strains lower shear stresses than this. Here, too, the lattice rotation of the alloy crystals is characterized by a substantial transgression of the symmetry position, thereby revealing the greater hardening of latent glide systems.

The extension of silver–gold solid solutions differs only comparatively slightly from the behaviour of the pure metals (235). Only at small deformations does the hardening of the solid solution remain noticeably below that of the pure metals. The lattice rotation, too,

is little affected by the formation of solid solutions.   The hardening of the latent octahedral system in this case exceeds that of the active system by about 12 per cent. at the most (at medium concentrations). The *copper–nickel* series of solid solutions appears to behave in a similar manner, *i.e.*, there is at medium concentrations more pronounced hardening of the latent octahedral system (236).

The significance of the *atomic arrangement* for shear hardening in plastic extension is evident from Fig. 115.   Whereas the cubic
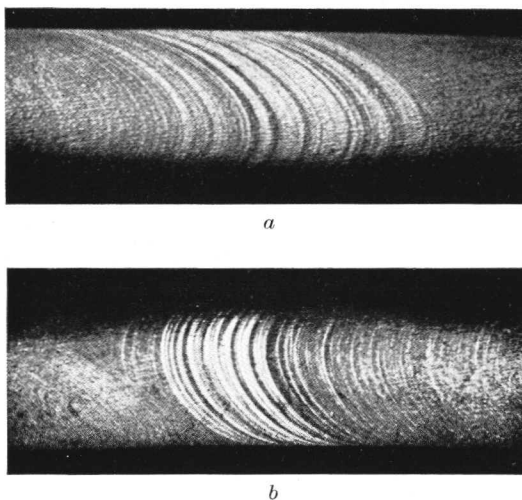

a

b

Fig. 114 (*a*) and (*b*).—Change of the Operative Glide System when Extending *a*-Brass Crystals (234). Appearance of New Bands.

(*a*) After the first change ;   (*b*) after the second change.

face-centred $AuCu_3$ crystal with disordered atom arrangement (quenched from 800° C.) shows the type of yield–stress curves characteristic of solid solutions (high initial shear stress and rather slow increase of the shear stress with increasing glide), in the case of ordered atom arrangement (produced by an ageing treatment of the crystal at 325° C.) the hardening curve rises steeply with increasing glide strain from a low initial value—as with pure metals.   The extent, too, to which the symmetrical position is transgressed, corresponds to this behaviour :   with disordered atomic arrangement it is twice that of the ordered arrangement.

The extension of *aged* copper–*aluminium* solid solutions (with

5 per cent. Cu) can also be represented by a mean hardening curve (238). However, the fracture of these crystals does not obey a condition of maximum load, as in the case of pure aluminium. Instead, as with Zn–Al solid solutions, fracture occurs in certain orientation ranges, without previous necking, along one or more nearly plane surfaces. The mechanism of this fracture is not known. The ultimate tensile stress, according to the initial orientation, lies between 33·0 and 45·5 kg./mm.², the elongation between 7 and 26 per cent.

FIG. 115.—Yield–Stress Curves of AuCu₃ Crystals (237).

(c) INFLUENCE OF TEMPERATURE AND TIME

The description of the effect of composition and structure upon the glide of metal crystals will now be followed by a discussion of the significance of temperature and speed of deformation. The dynamic properties have hitherto referred to room temperature and to a limited range of deformation speeds. However, it has been precisely the experiments conducted over a wide range of temperature and time which have led, if not to an explanation, then at least to a physical interpretation of crystal glide.

47. The Influence of Temperature and Speed upon the Start of Glide

The influence of the test temperature on glide has hitherto been widely investigated mainly with hexagonal crystals. It can be said of them in general that the sharp bend in the stress–strain curve at the yield point becomes more accentuated with rising temperature. It was seen in Section 43, in which we discussed the dependence of the stress–strain curve upon the orientation of the crystal, that the sharpness of this bend was decisively influenced also by the position of the glide elements. Where the glide elements are more or less transverse to the direction of tension, an initial
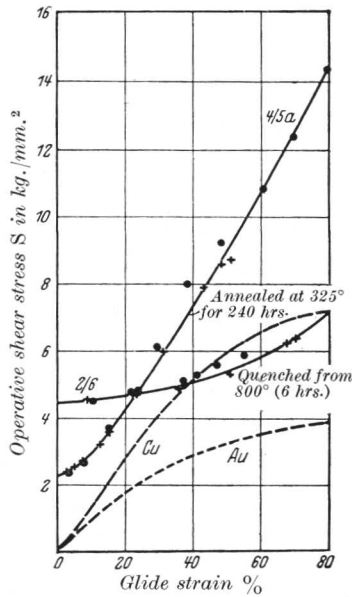
L

reduction in load with increasing extension is possible in spite of the shear *hardening* due to glide, whereas the same yield–stress curve
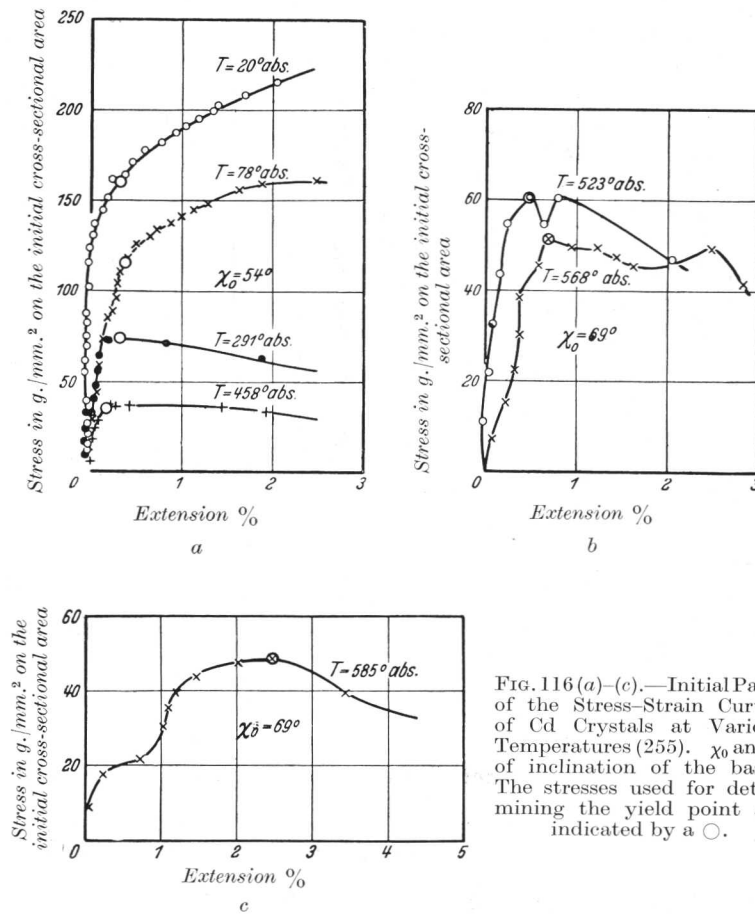

Fig. 116 (*a*)–(*c*).—Initial Parts of the Stress–Strain Curves of Cd Crystals at Various Temperatures (255). χ_0 angle of inclination of the basis. The stresses used for determining the yield point are indicated by a ○.

leads at once to a steep increase in load if the initial position of the glide elements is very oblique.

A characteristic example of the initial parts of stress–strain curves of several fractions of the same cadmium crystal, obtained at various temperatures, is found in Fig. 116, *a*, *b* and *c*. The test temperature ranged from 20° to 585° K., *i.e.*, to within 9° of the melting point. A stress–strain curve obtained with special equipment at still lower

temperatures down to 1·2° K. is shown in Fig. 117. It is seen that considerable plasticity persists even at these temperatures and that in particular the critical shear stress is of the same order of magnitude as at the other temperatures. This fact has also been confirmed for zinc crystals [(251), (253)].

Fig. 118 shows in diagrammatic form the relationship between critical shear stress and temperature. It will be observed that the shear stress of the basal glide system of hexagonal metals is only slightly affected by temperature. The increase in the shear stress
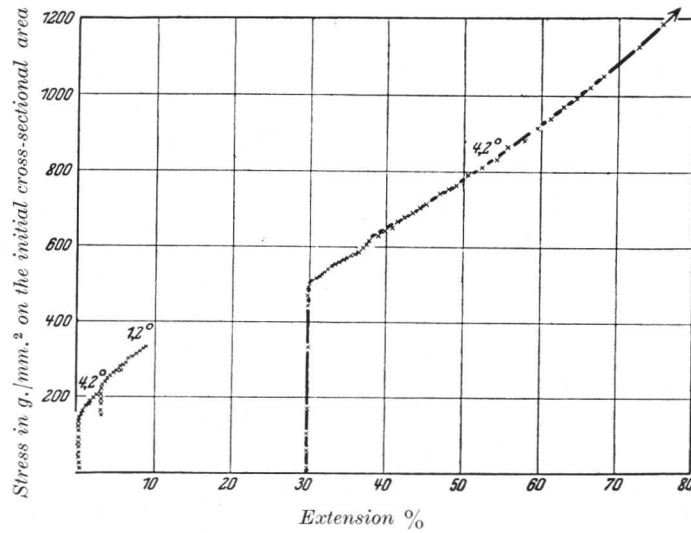

FIG. 117.—Stress–Strain Curve of a Cd Crystal at Low Temperatures (254).

of cadmium, for instance, is slightly less than four-fold, despite a reduction in temperature from just below the melting point to 20° K. A similarly slight dependence of S_0 on temperature has been observed with the rhombohedral bismuth (263) and tetragonal tin (262).

Although the experimental data available for very low temperatures are scanty (a serious deficiency, especially where cubic crystals are concerned), extrapolation down to absolute zero for zinc and cadmium can at least be attempted. It reveals that even at this temperature the low yield stress of the basal plane persists.

In the neighbourhood of the melting point there was in all cases a zone of approximately constant critical shear stress. A decrease

to zero value (that of the melt) must proceed very suddenly, at any rate for cadmium and bismuth; down to temperatures of 9° or 6° below melting point there is still the same value as in the range in which there is no dependence on temperature. In view of the temperature dependence of the solubility of impurities, their role in the above tests cannot at present be assessed.

It has already been mentioned that *aluminium* crystals show no sharp yield point at room temperature (see Section 40). This has also been observed for the temperature range −185 to 600° C. (257). However, the yield–stress curves shown in Fig. 128 indicate that
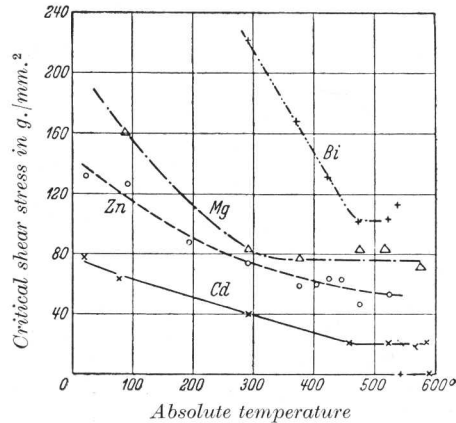

Fig. 118.—Effect of Temperature on the Critical Shear Stress.

for aluminium, too, there is only slight dependence of the start of glide on temperature. Whereas at 600° C. a shear stress of 100 g./mm.^{2} suffices for perceptible deformation, at −185° C. a stress of approximately 800 g./mm.^{2} is necessary to produce observable octahedral glide.

Having described the effect of temperature, we will now deal briefly with the effect of the speed of deformation on the critical shear stress. It will be explained later (Section 50) that, owing to their similar influence, time and temperature are probably very closely connected. It has been found that in general the yield point increases with increasing speed of testing. It is probable that the decisive factor is not the increase in the stressing speed within the range of purely elastic deformation, but the greater speed of deformation within the narrow range that lies between the elastic limit and

the yield point. Some idea of the magnitude of the influence of this speed can be gained from Fig. 119 in the case of cadmium crystals. A hundred-fold increase in the speed of testing leads, within a wide range of temperature, to an increase of the critical shear stress of 20–30 per cent. Fig. 119 reproduces, also, as a broken line, the curve which appears in Fig. 118; since it relates to crystals which were drawn from the melt much more slowly, it is not directly comparable with the other two curves.

It is difficult to give figures for aluminium crystals, owing to the absence of a sharp yield point in this case. From available tests
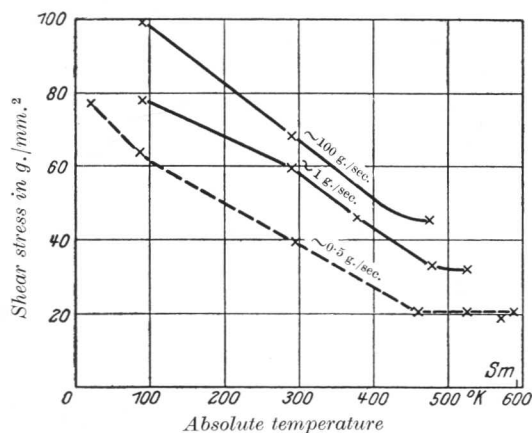

FIG. 119.—Dependence of the Critical Shear Stress of Cd Crystals upon the Speed of Stressing (Deformation) (252).

in which the test speed varied from 1 : 23,000 it appears that, in spite of this wide range, glide begins at approximately the same stress (256).

48. *Progress and Termination of Glide in Relation to Temperature and Speed*

An example of the energy needed to continue extension at various temperatures is shown in Fig. 120. It represents the extension of a cadmium crystal in stages at various temperatures. The change in temperature was always effected after removal of the load from the crystal. It will be noticed that the relatively slight dependence of the critical shear stress upon temperature in the initial unworked state persists at larger glide strains. On the other hand, the work needed

to continue the extension is strongly dependent upon temperature. Whereas at room temperature and even more so at elevated tempera-
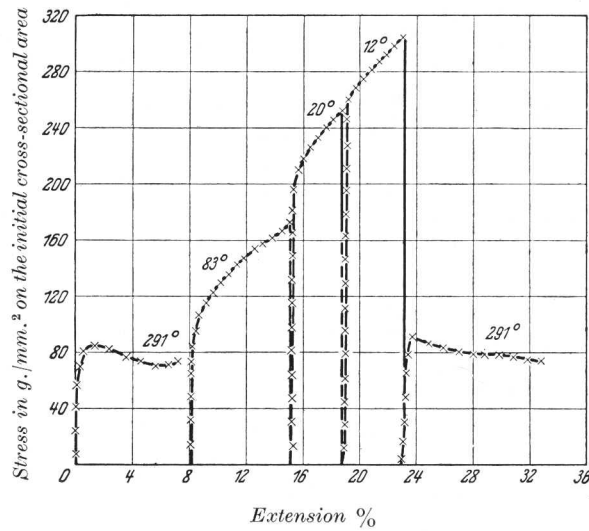

FIG. 120.—Stress–Strain Curves of a Cd Crystal at Various (Absolute) Temperatures (254).

tures stretching can take place initially with a falling load (however, here too the shear stress in the operative basal glide system increases) a very considerable increase in load is necessary at low
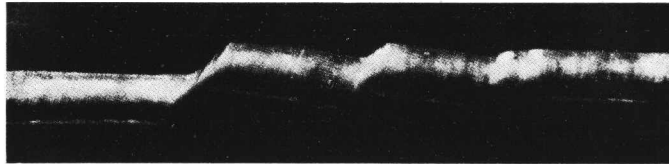

FIG. 121.—Local Glide of a Cd Crystal Extended at 250° C. (255).

temperatures. Consequently at low temperatures stretching usually proceeds much more uniformly along the whole crystal than at temperatures at which, owing to the reduction in stress, it is confined initially to the weakest portions of the crystal which experience the first extension (Fig. 121).

As a rule the glide bands become coarser as the temperature rises (Fig. 122 compared with Fig. 35a). Quantitative statements regarding the mean thickness of the glide packets are not available. Difficulty arises from the circumstance that, with increasing magnification, new glide lines becomes visible.   In the case of regular zinc
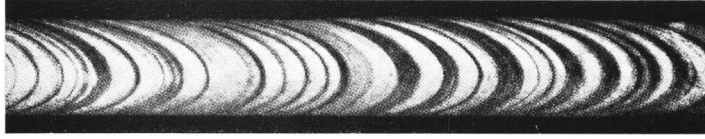

FIG. 122.—Zn Crystal Extended at 300° C. (252).

polyhedrons which were grown from vapour, values were obtained which fluctuated around 0·8 μ., or a multiple thereof (267).   Tin crystals usually show glide bands; sometimes, however, they appear perfectly smooth in spite of substantial deformation (extension up to ten times the initial length); the thickness of the glide packets must then be below microscopic resolution (261).   Reduction of the
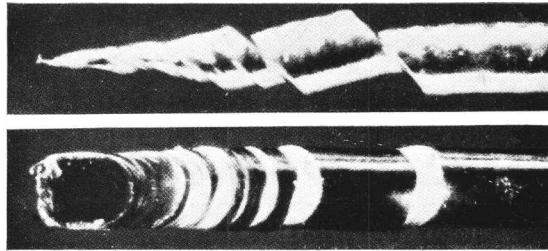
FIG. 123.—Formation of Coarse Glide Packets and of a Shear Fracture in the Course of Very Slow Extension of a Sn Crystal (see 260).

speed of straining, like an increase in temperature, produces coarse glide bands. Fig. 123 contains two illustrations of zinc crystals which have been stretched extremely slowly.

The yield–stress curves obtained for *hexagonal* metal crystals at different temperatures are shown in Fig. 124. They reveal a very pronounced influence of temperature on the shear hardening of the basal plane. The hardening coefficient derived from the slope of the yield–stress curve changes 400-fold within the temperature range

investigated. This dependence is most marked at intermediate temperatures; it decreases rapidly at both high and low tempera-
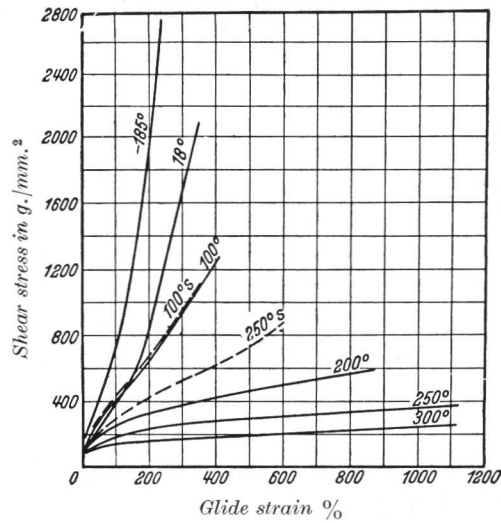
(a) Magnesium (249).
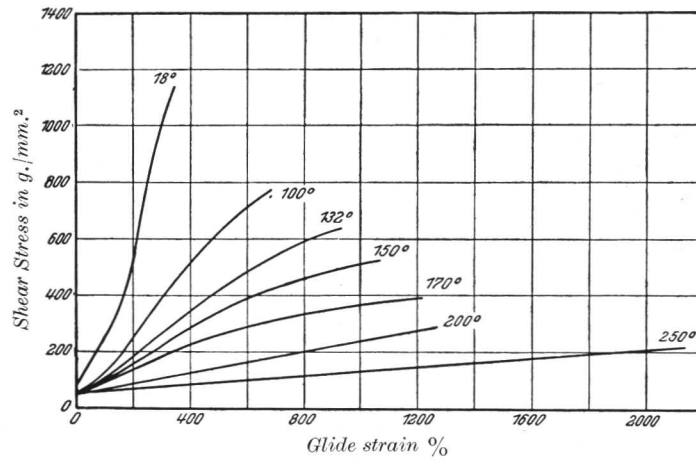
(b) Zinc (253).

tures, and becomes insignificant where these are extreme. For very low temperatures this is apparent from Figs. 117 and 120 (cadmium crystals).

The amount of basal glide that can occur at different temperatures, which in the case of magnesium is limited by shear fracture along the basal plane, or by the start of pyramidal glide, and in the case of zinc and cadmium by secondary glide in twin lamellæ (see Section

FIG. 124 (a)–(c).—Yield–Stress Curves of Hexagonal Crystals as a Function of Temperature. Speed of Stressing before Reaching the Yield Point: 80–100 g./mm.²/min. Discontinuous Curves Obtained at a Stressing Speed which was about 100 Times Greater.

Glide strain %
(c) Cadmium (252).
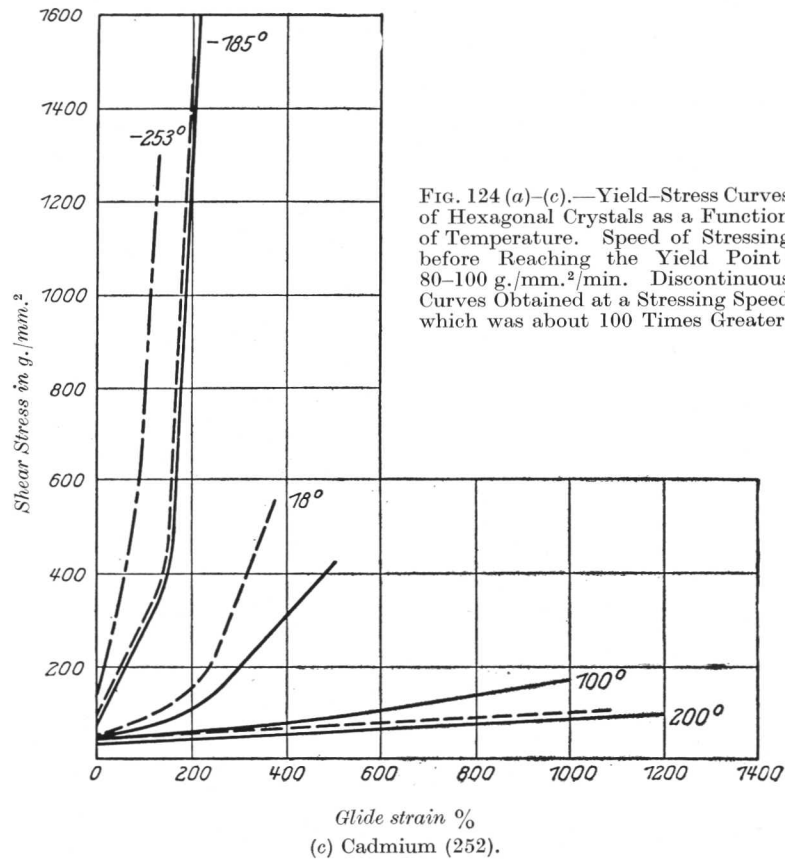
44), is again well expressed by a limiting shear stress which is independent of orientation. Thus within a wide range of temperature the mean yield–stress curves for basal glide are characterized not only by their starting point and slope, but also by the point at which they end. The relationship between temperature and the limiting values for shear stress and glide strain is shown in Fig. 125. In the temperature range investigated extension by basal glide increases

by an order of magnitude, while at the same time hardening decreases equally rapidly. This entails even with the flattest yield–stress

(a) Limiting shear stress.   (b) Limiting glide strain.
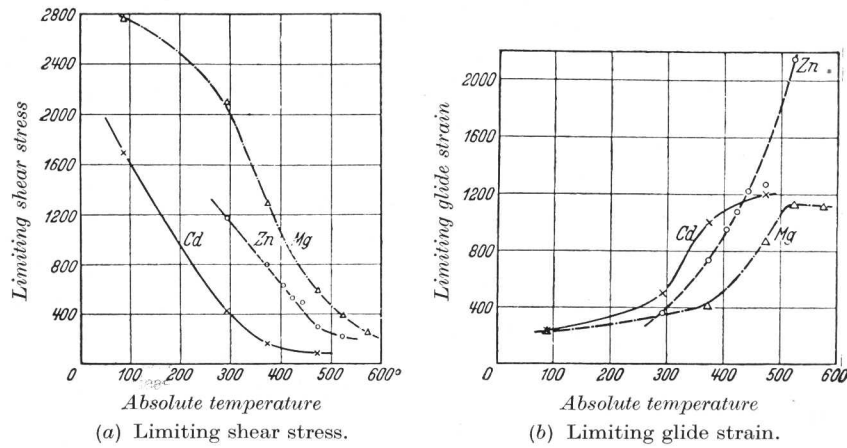
FIG. 125 (a) and (b).—Termination of the Basal Glide of Hexagonal Crystals as a Function of Temperature.

curve an increase in the shear stress during extension of three times the original value.

While the slope of the yield–stress curve and of the limiting glide

Absolute temperature
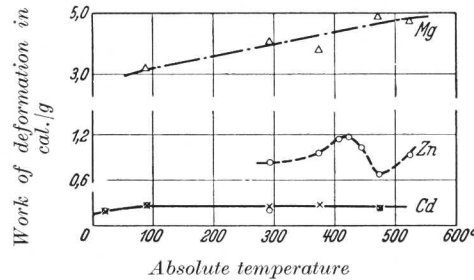
FIG. 126.—Work of Deformation Required for Basal Glide as a Function of Temperature.

(For Cd : ✕ = slow fracture, in about 20 min.; ○ = rapid fracture in about 10 sec.)

strain and shear stress markedly depend upon temperature, the *work of shear* for the three hexagonal metals for basal glide is found to be *approximately* independent of temperature (Fig. 126). Zinc crystals, however, show a flat maximum at 150° C. It is not known

for certain whether this is bound up with a possible deviation from pure basal glide which may occur in this range of temperature. The magnitude of the mean shear energy amounts to 4·4 cal./g. for magnesium (that is 18 times the specific heat, or 1/10th of the latent heat of fusion); for zinc it is 0·97 cal./g. (11 times the specific heat, or 1/24th of the heat of fusion), and 0·24 cal./g. for cadmium (4 times the specific heat, or 1/44th of the latent heat of fusion). This purely empirical finding can hardly represent a limiting condition, because in the course of deformation the greater part of the added energy is transformed into heat and is therefore no longer present in the crystal at the conclusion of glide. It would be more reasonable to suppose a *saturation* with internal energy, which in the cases described here, of course, would have to amount always to the same fraction of the total work of deformation. (For further details see Section 60.)

In Fig. 124, *a* and *c*, the dependence of the yield–stress curve on time at different temperatures is also shown by curves obtained at a one hundred-fold speed of deformation. It is found that within the range in which the yield–stress curve is very dependent upon temperature there is also a marked dependence upon speed; in the case of magnesium, for instance, the shear stress after 600 per cent. glide at 250° C., in the crystal that has been stretched rapidly, is three times greater than in that which has been stretched slowly.[1] Within the range in which the yield–stress curve is less dependent upon temperature the speed of deformation is also without considerable influence. The work of shear, which is independent of temperature, is also largely independent of the speed of deformation, as is seen in the case of cadmium in Fig. 126.

In the extension of zinc and cadmium crystals represented in Fig. 124, *b* and *c*, the primary basal glide is restricted by the start of a new basal glide within the twin lamellæ formed towards the end of primary glide. If the temperature exceeds the maximum values given in the diagram, then the normal course of basal glide is disturbed by recrystallization during the extension. At temperatures above 300° C., cadmium crystals break transversely without considerable elongation. With zinc crystals, decrease of temperature leads at −80° C. to pronounced reduction of the ductility and to the appearance of cleavage fracture along the basal plane—a phenomenon

[1] Great importance therefore attaches to the constancy of test speed in certain temperature ranges if the tests are to be comparable. In accordance with the crystallographic process of glide, the value to be maintained constant is the rate of glide strain which is independent of the initial position and of the change in orientation in the course of stretching (256).

which is observed at all lower temperatures. (For further details see Section 53.)

Only approximate details are available for *tin* crystals. While the slope of the yield–stress curve remains more or less constant, the limiting glide strain falls appreciably with rising temperature (up to 200° C.). In this case the range within which the hardening
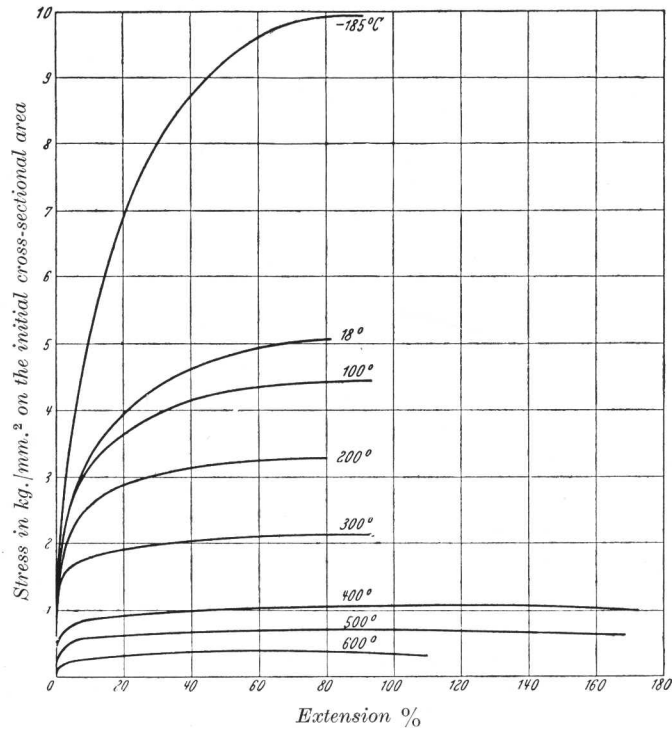
Fig. 127.—Stress–Strain Curves of Al Crystals in Relation to Temperature (257).

coefficient is only slightly dependent on temperature is reached already between 20° and 200° C. (262).

The relationship between temperatures and the stress–strain curves of *aluminium* crystals is given in Fig. 127. From these it will be seen that hardening is greatly reduced as temperature rises, but that the total extension between −185° and 300° C. remains noticeably constant. This clearly distinguishes aluminium crystals from the hexagonal crystals, which, in general, experience sub-

stantial extension at higher temperatures. The increase in the extension of aluminium crystals at 400° and 500° C. is followed at 600° C. by a substantial reduction, due to the premature elimination of primary octahedral glide by the emergence of a new (cubic) glide. Recrystallization was observed in this case only at the highest temperatures, and even then only at the apex of the extension cone in the neck.[1]

The relationship between temperature and the yield–stress curve based on primary octahedral glide (glide along the system which first becomes operative) is shown in Fig. 128. While at the start of glide the influence of temperature is only small, here, too, the effect of temperature on the increase of the shear stress becomes very marked as the strain increases.

The influence of the *test speed* on shear hardening has been studied at room temperature only (256). The yield–stress curve obtained in dynamic stressing at a 23,000-fold speed (450 per cent. elongation per second) exhibits shear-stress values which are approximately 16 per cent. higher than for the static curve. A test which was carried out at an exceptionally reduced speed of loading (test period : 234 days) is described in (258).

Particulars of the qualitative influence of time on glide are available also for α-iron (259), tin (261) and bismuth (250). In all cases an increase of the speed of testing leads to more pronounced hardening and to a reduction of plasticity. This is particularly noticeable
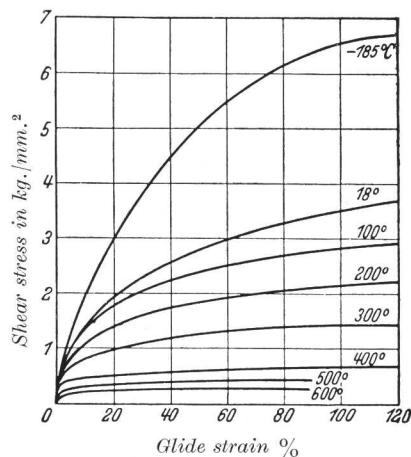
FIG. 128.—Yield–Stress Curve of Al Crystals in Relation to Temperature (257).

[1] The lattice position in which cubic glide undoubtedly starts enables us to estimate its yield stress relatively to that of an octahedral system, as was mentioned in Section 42. It is true that this will apply to extended crystals only; the corresponding ratio for the intact initial crystal need not be the same. This estimate is based on the fact that, in the end position [112] towards which octahedral glide tends, cubic glide occurs, and leads to a new end position [111]. Consequently, the shear stress in the cube plane in the direction of a face diagonal exceeds that of an octahedral glide system by only about 15 per cent.

in the case of the tin crystal, owing to the simultaneous operation of several glide systems in rapid stretching. The same effect is produced by a reduction of the test temperature to −185° C.

49. *Crystal Recovery*

In the previous section we dealt with the significance of temperature and time in the plasticity of crystals, and we gave particulars of the yield–stress curves in relation to temperature and speed of deformation. In so far as an influence exists, it takes the form of a reduction of the hardening with rising temperature or diminishing speed of deformation, for a given deformation. The experiments
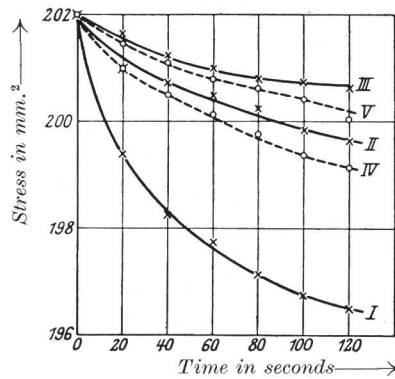
FIG. 129.—Flow Curves of a Sn Crystal after Hardening and Recovery (261). Between Tests III and IV the Unloaded Crystal was Heated for 1 Minute at 60° C.
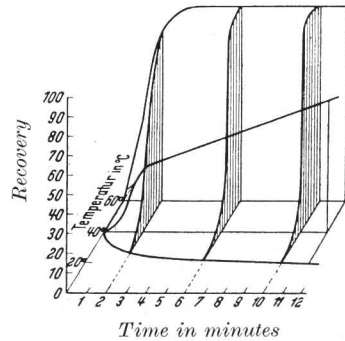
FIG. 130.—Recovery Diagram of a Sn Crystal (250).

which we are about to describe, and which have led to the conception of " crystal recovery ", illustrate even more directly the softening effect of temperature and time [(264), (266), (265)]. In addition, they clearly reveal the close connection between these two factors.

Fig. 129 contains flow curves of a tin crystal after repeated stretching at the same stress, and after a rest of 1 minute at 60° C. The curves were recorded in a Polanyi filament-extension apparatus (see Fig. 75). All extensions took place below the yield point of the crystal. The initial reduction in the mean speed of flow reveals the hardening which accompanies the deformation ; while the weakening effect of intermediate annealing is shown by the resulting increase in

flow. This weakening, which is not a result of recrystallization or a change in orientation of the crystal, can be modified within a wide range by changing the conditions of annealing. The measure of the softening (e) is obtained from $e(\%) = \dfrac{v'_0 - v_e}{v_0 - v_e} \cdot 100$, where v_0 represents the original flow speed, v_e the flow speed of the hardened crystal, and v'_0 that of the recovered crystal. In the limiting case of $v'_0 = v_e$ there is no recovery; if, on the other hand, the original flow speed is again reached ($v'_0 = v_0$) recovery has been 100 per cent. A three-dimensional diagram, based on such tests, which shows the recovery of zinc crystals as a function of time and temperature, is given in Fig. 130. Recovery can be considerable even at room temperature over sufficiently long periods; it becomes perceptible after 10 minutes and amounts to about 50 per cent. after 20 hours. Analogous flow tests proved that *bismuth* crystals also possess a marked capacity for recovery (250).

An even more striking demonstration of the softening effect of time and temperature is shown in Fig. 131, which shows the exten-
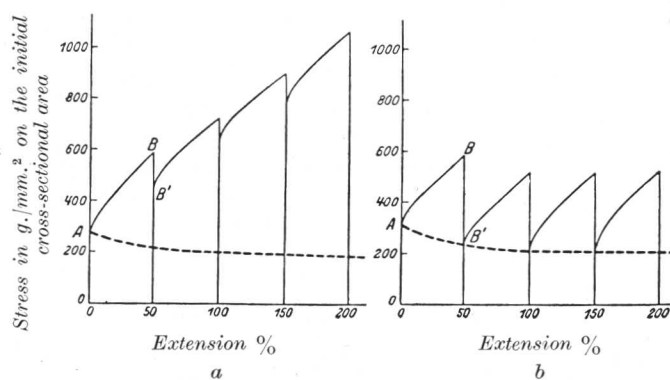
FIG. 131 (*a*) and (*b*).—Extension of a Zn Crystal; Tests Interrupted for 30–40 sec. in (*a*), and for 24 hours in (*b*), to Permit of Recovery.

sion curves of a *zinc* crystal. After each extension of 50 per cent. the load was removed from the crystal for 30–40 seconds in the one case, and for 24 hours in the other. The softening which occurred during the interval is expressed in the reduction of the stress necessary for further extension (from B to B'). The broken curves in the diagrams represent the extension curves calculated theoretically on the assumption of constant shear stress of the glide system (formula

43/1). It will be seen that the *short* pause resulted in only a slight softening, whereas in the 24-hour interval the strongly increased yield stress of the basis reverted to its original value. Therefore the crystal recovered completely during this period. The measure of recovery in these tests is given by the expression

$$e(\%) = \frac{S_1 - S'_1}{S_1 - S_0} \cdot 100,$$

S_0 being the initial yield stress of the unstressed crystal (Point A), S_1 the yield stress at the close of the first stage of extension at 50 per cent. (Point B), and, finally, S'_1 the yield stress of the pre-worked and recovered crystal (Point B′).

The occurrence of recovery in *aluminium* crystals is clearly apparent in creep tests (flow under constant load) at elevated temperature (250° C.). In this case the extension of the crystals proceeds in three different stages. Upon application of the load the crystal at first flows rapidly, and in so doing forms numerous glide bands; then, as hardening begins, there follows a stage of small deformation in which only a few new glide bands occur, while finally, as a result of crystal recovery, rapid flow occurs which leads to fracture (258).

Tungsten crystals, too, exhibit very distinct recovery after suitable heat treatment (at least 10 minutes at 2100° C.). This was revealed by the considerable reduction in the ultimate tensile stress of crystals that had hardened by drawing through diamond dies. The original softness of the crystals was restored by this heat treatment, indicating that here, too, the shear stress in the glide systems had been reduced (266).

Although the direct experimental investigation of crystal recovery has been limited to a few metals only, there can be no doubt that we are faced here with a very general phenomenon which is technically of great importance. In summing up it may be said that the essential feature of crystal recovery is that, while preserving the lattice, it *continuously* reduces, during heat treatment, the mechanical properties of the work-hardened crystals approximately to the initial values. Consequently, crystal recovery contrasts sharply with recrystallization which involves the formation of new grains and reduces the properties of the hardened metals down to their original values *in stages* (cf. Section 65).

Hitherto the weakening effect of recovery has been discussed in relation to the resistance of the crystals to plastic deformation; it will be shown later that recovery influences also their tensile strength.

50. *Glide Regarded as the Superimposition of Recovery upon an Athermal Fundamental Process*

In the previous sections the influence of temperature and time on the progress of glide have been discussed. The main facts which have been established, chiefly with hexagonal crystals, can be summed up as follows : the critical shear stress which characterizes the start of glide is only slightly dependent upon temperature (and the speed of loading). There is no reason why it should not be assumed that the shear stress in the glide system at absolute zero is of the same order of magnitude as at elevated temperatures.

In marked contrast to this limited dependence on temperature of the initial yield stress, the increase in the shear stress in the course of glide (hardening) depends very strongly on the temperature, at any rate within certain ranges. At very low temperatures both the start and the slope of the yield–stress curve are practically independent of temperature. On raising the temperature a range is reached in which the hardening is appreciably reduced as the temperature increases; and finally the hardening curve, which now rises only very slightly, again becomes almost independent of temperature.

The speed of deformation has a very similar influence on the yield–stress curve : there is an insignificant effect at very low temperatures and near the melting point, but a very strong influence exists in the intermediate range.

The following physical description of crystal glide is suggested by these facts (251). The basic process is athermal, *i.e.*, independent of time and temperature. It operates undisturbed at very low temperatures, and finds expression in a characteristic function of the material, the yield–stress curve, which indicates that the low yield stress of the glide system of the unworked crystal greatly increases with increasing deformation. Upon this basic process, which is independent of temperature, there is imposed, as the temperature rises, a thermal-softening process : *crystal recovery*. In general, therefore, two fundamentally distinct processes are simultaneously involved in the deformation of a crystal by glide : the one concerns the actual crystal and is structurally determined; the other is based on the thermal vibrations of the lattice. As temperature rises, the thermal recovery increasingly cancels out the hardening caused by glide, leading ultimately in extreme cases to a deformation which is free from hardening and which takes place at a constant shear stress of the glide system.

From the above it will be immediately apparent that an increase

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of time, *i.e.*, a reduction of the speed of testing, has the same effect upon the softening process as an increase of temperature.   This is a plausible explanation of the influence which speed exerts on the yield–stress curve.

C. THE DYNAMICS OF TWINNING

51. *The Condition of Twinning*

Whereas glide could be characterized dynamically by the critical shear stress and the yield–stress curve, similar quantitative principles governing mechanical twinning are not yet available.   As already explained, the formation of a deformation twin represents, contrary to glide, a process that is also macroscopically discontinuous. Fig. 132*a* illustrates this by the extension curve of a cadmium crystal (Polanyi filament-stretching apparatus), which clearly reveals the formation of twin lamellæ (accompanied by audible crackling) resulting from discontinuous changes of load.   The twins themselves are usually narrow lamellæ which often do not traverse the entire crystal; and from these lamellæ initially only occasional small bands develop, which sometimes widen as the stress increases, or are replaced by the formation of new twins. During the formation of a twin lamella a further increase of load is unnecessary.   It will suffice, therefore, for a dynamic characterization of twinning if an account is given of the conditions which govern its *initiation*.   The reasons why we are unable at present to make any statement regarding this " critical " state are to be sought on the one hand in the very pronounced sensitivity of twinning to inhomogeneities within the crystal, as a result of which the deformation twins always develop in a more or less extended stress range. On the other hand, the attempt to formulate quantitatively the conditions which govern twinning is greatly hampered by the circumstance that, as a rule, the crystals also exhibit glide, in which event, of course, twinning cannot be studied in the intact initial state but only in an already hardened state.   An example of this will be found in the extension curves of variously oriented cadmium crystals shown in Fig. 132, *b* and *c*.

The few indications at present available for the initial law originate from alternating-torsion tests.   These tests, which were carried out on zinc and bismuth crystals, would appear to indicate that a single initial law based on a limiting stress, such as governs glide, does not exist for mechanical twinning [(268), (269)].   It is thought that twinning obeys an energy condition for which the path

removal of the stress, has been investigated on zinc and tungsten crystals (288). The crystal wires were twisted by certain angles (5–10° in the case of 10-cm.-long zinc crystals, 180° for the 50-cm.-long tungsten crystals) and then released. Whereas the tungsten crystals immediately regained their initial position, the zinc crystals retained a substantial permanent torsion. In both cases, however, contrary to comparative tests carried out with polycrystals, the final position was already reached 1 minute after removal of the load. No further changes occurred during the investigation, which extended over 36 hours, whereas in the case of polycrystalline wires over the same period there was a subsequent reverse deformation amounting to 40 divisions on the scale (corresponding to an angle of torsion $\sim 2°$). These experiments show that the torsional after-effect of metal single crystals, if it exists, is very much smaller than with polycrystalline material.

Elastic *hysteresis* means that the shape of a solid is dependent not only upon the instantaneous value of the stress but also upon the stress–strain history of the material. So far investigation of the effect in single crystals has been limited to aluminium (291). Tensile loads were applied to the crystals slowly and then slowly removed, and the resulting changes in length were observed by means of the Martens mirror apparatus. The stresses were kept within the range $0 \cdot 028$–$1 \cdot 43$ kg./mm.^{2}. The total extension after a large number of load cycles amounted ultimately to $2 \cdot 2$ per cent. The results of the tests can be summarized as follows: plastic extension already occurred at the first application of load, even within the minimum stress range of $0 \cdot 028$–$0 \cdot 144$ kg./mm.^{2}; therefore a range of a purely elastic deformation was not observed. Repeated loading and unloading, after the initial occurrence of permanent deformation, led ultimately to closed hysteresis loops similar to those observed with polycrystals. The existence of these loops is not compatible with a homogeneous elastic deformation of the lattice.

The Bauschinger effect consists in the observed fact that, after a test bar has been deformed, its resistance to deformation (elastic limit, yield point) is greater for further deformation in the same direction than for deformation in the reverse direction (289). In certain circumstances, even a weakening can be observed in the reverse direction. For single crystals the Bauschinger effect was studied on α-brass (72 per cent. copper). Cylindrical bars $9 \cdot 2$ mm. in diameter were subjected to tension–compression tests, the changes in the 10-mm. gauge length being observed by means of a Martens mirror apparatus. The results obtained with the single crystal

M. Polanyi (287) created the conception of strain strengthening and contrasted it with shear hardening. He observed strain strengthening on deformed tungsten and rock salt crystals (286). Only with rock salt is the surface of fracture a smooth plane. The strengthening of tungsten is assessed on the basis of the strength related to the final section.

Finally, it may be mentioned that in deformed crystals strain strengthening, like shear hardening, can be gradually removed by *crystal recovery*. Figures relating to this phenomenon, for the fracture of zinc crystals on the prismatic-plane type I, are contained in Table XVII. After extension at room temperature, and before fracturing at −185° C., the crystals were heated for 2 minutes at 80° C. This produced a tensile weakening of 30–40 per cent. without recrystallization of the crystal. Just as deformation produces both strain hardening and strain strengthening, recovery leads to both a softening and a weakening of the crystal (decrease of critical shear and normal stress).

E. AFTER-EFFECT PHENOMENA AND CYCLIC STRESSING

Hitherto we have examined the behaviour of metal crystals under substantial plastic deformation. We will now describe those closely related phenomena which occur within a range of small deformation upon removal of the load or change of the direction of stress. In the first place it is a question of *after-effect* and *hysteresis* which, although usually termed elastic processes, must no doubt be regarded essentially as plastic. The Bauschinger effect will also be discussed in this context since it is probably due to the same causes. The available material relating to these phenomena in metal crystals is at present confined to a few experiments. A close enquiry into this behaviour will, however, certainly contribute greatly to our knowledge of crystal deformation and hardening.

Subsequently we shall describe investigations carried out on metal crystals in regard to the technically very important property of fatigue. In discussing these experiments we shall deal first with the external crystallographic features of the crystals resulting from cyclic stressing (deformation phenomena, crack formation), and then with the changes which take place in the mechanical properties as the stress increases.

55. *After-effect and Hysteresis.   Bauschinger Effect*

The elastic *after-effect*, which consists of an " after-shortening ", *i.e.*, establishment of equilibrium length as a function of time after

cleavage planes are independent of the glide strain, as is more or less the case at this temperature, then the limiting glide strain which is achievable for crystals of various initial orientations is obtained by intersection of the series of curves with the horizontal straight lines at distance N_B or N_P from the abscissa axis. It will be seen that where the initial positions of the *basal plane* are oblique, its critical normal stress (180 g./mm.^{2}) is obtained only after considerable glide. Fracture would have to be preceded by a very substantial deformation. It has been found, however, that even after relatively small deformations the critical normal stress (1800 g./mm.^{2}) for the *prismatic plane* is reached. These crystals fracture therefore by cleavage along the prism plane.

Beyond an χ_0 angle of approximately 25° this prismatic cleavage is then replaced by fracture along the basal plane. Whereas formerly the glide strain increased with increasing χ_0, it now begins to diminish. For a χ_0 angle corresponding to the equation $tg\chi_0 = \dfrac{N_0}{S_0}$ the critical normal stress is already reached at the yield point, so that for all χ_0 angles $>$ arc $tg\dfrac{N_0}{S_0}$ ($\sim$58°) brittle fracture of the crystals occurs.[1] The critical normal stress was here assumed constant; however, even if it depended on the preceding deformation the examination would follow a similar course.

Although the interpretation of the fracture process at low temperatures as one of shear fracture is contradicted by the appearance of the fracture surfaces of zinc crystals, it should be pointed out that this view is also untenable for another reason. If the onset of fracture resulted from the limiting shear stress, then this would always have to operate (owing to the yield–stress curve which is valid at low temperatures) upon reaching a certain point on this curve, that is, at a *constant limiting glide which is independent of orientation*. This does not agree with experimental results.

Apart from this case of the zinc crystal, which made possible an investigation of strain strengthening of an operative glide plane (the basal plane) and of a plane which crossed this (prismatic-plane type I), there are no other examples of the strain strengthening of metal crystals by glide. The question of the influence of mechanical twinning upon the normal strength of cleavage planes is still quite obscure.

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[1] It should be noted that all curves in Fig. 143a have the straight line $N = k$ as asymptote. Whereas sooner or later fracture occurs in all orientations for values of the critical normal stress $N < k$, for $N > k$ a fracture about the plane under consideration is possible within only a restricted range of orientation, and then only if there has been no previous glide.

considerable basal glide at room temperature has a very pronounced influence on the normal strength of the prismatic plane (determined at $-185°$ C. immediately after stretching). The varying amounts of extension were achieved through the use of crystals of different initial orientations. Table XVII gives the fracture-strength values obtained, to which the critical normal strengths of the prism planes are proportional owing to the very similar lattice position in stretched crystals. The increase of the strain strengthening of the prism plane with increasing extension is very clear.

From a knowledge of the behaviour of the critical normal stresses for the basal and prismatic planes it is possible to understand tensile
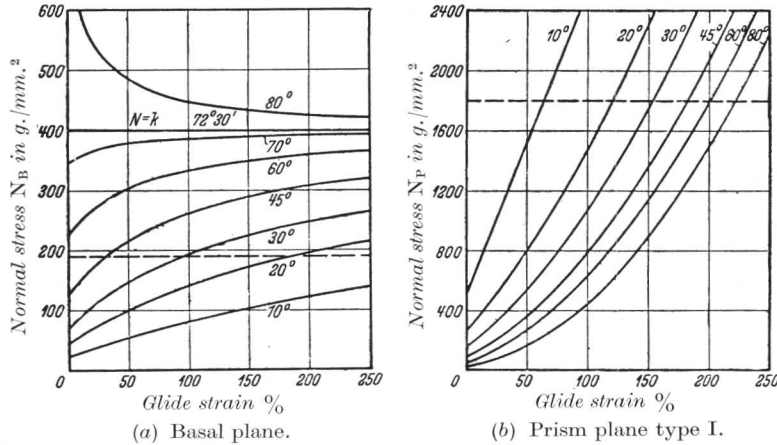

(a) Basal plane.   (b) Prism plane type I.

FIG. 143 (a) and (b).—Theoretical Course of the Normal Stress when Extending Zn Crystals of Various Orientations at $-185°$ C. The Initial Angle between Basal Plane and Longitudinal Direction is Indicated for the Respective Curves.

tests with zinc crystals, and, in particular, the deformation which precedes fracture at those temperatures where fracture occurs in one of the cleavage planes. In Fig. 143 is plotted the increase, during the tensile test, of the normal stress on the basal plane and on the prismatic-plane type I which lies most transversely, deduced from the yield–stress curve for crystals of various initial orientations at $-185°$ C.[1] If the critical normal stresses (N_B and N_P for the two

[1] N_B was calculated with the aid of the relationship

$$N_B = \frac{S_0 + ka}{a + \cot g\ \chi_0}\ (S_0 = 126\ \text{g./mm.}^2,\ k = 400\ \text{g./mm.}^2),$$

which is easily deduced from the formulæ (40/2) and (43/3): while N_P was calculated from the equation $N_P = \cos^2 30\ (S_0 + ka) \cdot (a + \cot g\ \chi_0)$.

N

The effect of this deformation on the critical normal stress of the basal plane is seen from Fig. 142. Although the results of the individual tests are scattered, the following facts do at least emerge. As was pointed out in Section 53, at $-185°$ C. the influence of preceding glide is only slight; at $-82°$ C. there is a marked increase, at $-253°$ C. a pronounced decrease in the critical normal stress with increasing glide, which in this case just exceeds 20 per cent. This shows that, according to the temperature, the normal strength of the glide plane can be increased or reduced by preceding glide. The influence of strain hardening, due to deformation, on the critical normal stress of cleavage planes ("strain strengthening")

TABLE XVII

Strain Strengthening and Recovery of Zinc Crystals (279)

Extension, %.	Fracture strength at $-185°$ C.		Recovery, %.
	Immediately after extension, kg./mm.².	After recovery for 2 minutes at 80° C., kg./mm.².	
50	2·6	—	—
100	3·2	—	—
160	5·5	3·4	38
300	7·3	—	—
350	8·2	—	—
380	8·4	6·2	26
400	9·6	5·4	44
500	9·8	—	—

can therefore be negative, in which case it represents a weakening. This contrasts remarkably with the effect of strain on the yield stress of glide systems. In the latter case there is nearly always a hardening (shown by the yield–stress curve) which increases with decreasing temperature and finally reaches a limiting value (see Section 48). If, at the yield point of zinc crystals, normal stresses of about 500 g./mm.² on the basal plane at 20° C. do not lead to fracture, this may perhaps be attributed to a strain strengthening which is caused at this temperature by the slight extension which occurs before the yield point is reached.

With the prismatic cleavage plane of zinc crystals as well as the basal plane there appears at $-185°$ C. to be no marked influence of the basal glide which precedes fracture. For extensions between 11 and 48 per cent. the critical normal stresses scatter unsystematically between 1330 and 2130 g./mm.² (278). On the other hand,

stresses. This is partly due to the difficulty of ensuring correct gripping of a cylindrical specimen when testing brittle cleavable crystals. The magnitudes of the normal strengths of the cleavage
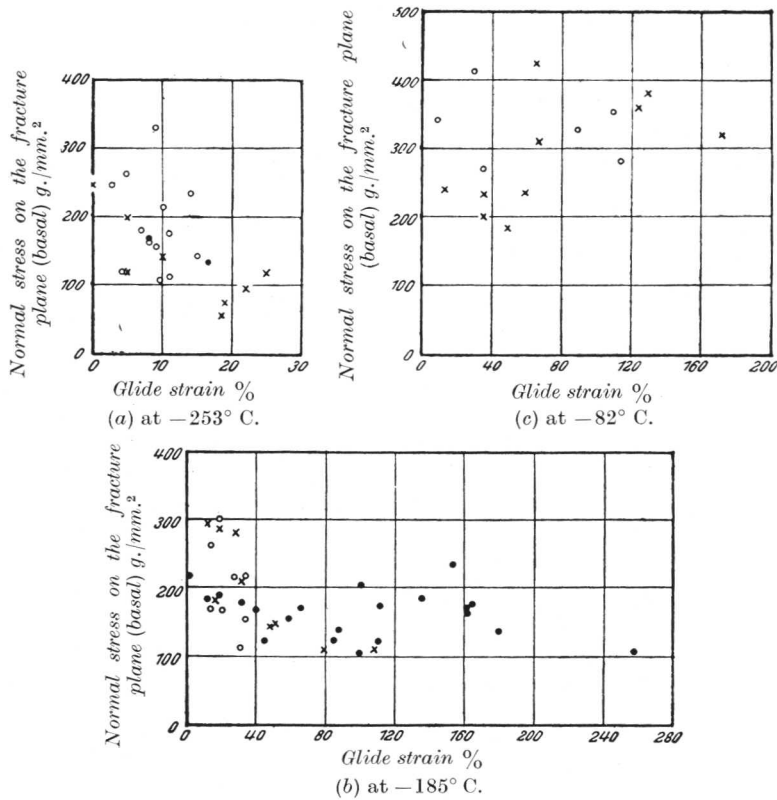

(a) at −253° C.

(c) at −82° C.

(b) at −185° C.

Fig. 142 (a)–(c).—Influence of Previous Glide on the Critical Normal Stress on the Basal Plane of Zn Crystals at Various Temperatures (281).

planes are of the same order as those of the shear strengths of the principal glide systems (Table IX).

Further tests are necessary if the significance of the critical normal stress in a quantitative description of *cleavability* is to be elucidated.

54. *Influence of Previous Deformation upon the Critical Normal Stress. " Strain Strengthening "*

It was pointed out in the previous section that the fracture of metal crystals on cleavage planes is often preceded by plastic deformation. This is specially obvious in the case of zinc crystals.

the cleavage plane.   The value obtained for the critical normal stress
of the rhombohedral plane, which is more than double that of the
(111) plane, can therefore be taken only as an approximation.

Only scanty experimental information is so far available regarding
the dependence of the critical normal stress on temperature, and in
any case we have seen that such dependence can be only slight.
Thus the tensile strengths for bismuth at 20° and −80° C. can be
represented by the same value of the critical normal stress of the
basal plane (Fig. 137).   And even for the zinc crystal there appear
to be no noticeable differences in the basal normal stress within the
range −80° to −253° C. (Section 54).

TABLE XVI

Critical Normal Stress of Metal Crystals

Metal.	Method of producing the crystal.	Cleavage plane.	Tempera-ture, ° C.	Critical normal stress at fracture, kg./mm.².	Litera-ture.
Zinc (0·03% Cd) .	Drawn from the melt.	(0001)⎱ (10Ī0)⎰	−185	0·18–0·20 1·80	(278) (281)
Zinc + 0·13% Cd		(0001)	−185	0·30	⎱ (280)
Zinc + 0·53% Cd		(0001)	−185	1·20	⎰
Bismuth . .	Drawn from the melt and soli-dified *in vacuo*	(111)	+ 20 − 80	⎱ 0·324 ⎰	(284)
		(111)⎱ (11Ī)⎰	+ 20	0·29 0·69	⎱ (285)
Antimony . .		(11Ī)	+ 20	0·66	⎰
Tellurium . .	Solidified slowly	(10Ī0)	+ 20	0·43	(283)

Scarcely anything is known of the influence of metals in *solid
solution* on the critical normal stress.   In the case of zinc crystals
alloyed with cadmium there appears to be quite a substantial
increase in the fracture strength (280).

A summary of the critical normal stresses so far measured is
given in Table XVI.   In addition to the examples already mentioned,
zinc (fracture along basal and prismatic planes, alloying effect),
bismuth (fracture along basal and rhombohedral planes, influence
of temperature), and tellurium, the table contains an approximate
value for the critical normal stress of the rhombohedral plane of
antimony.   It should be particularly noted that the values for the
normal stresses are less reliable than those for the critical shear

glide, constancy of the shear stress can at present be described only as much more probable than constancy of strain, in the case of brittle fracture of metal crystals the balance is clearly in favour of the constant-stress law.

The zinc crystal affords also an opportunity for determining the critical normal stress of the second-best cleavage plane, the prismatic-plane type I, which appears as cleavage plane when the basal positions are very oblique. At $-185°$ C. a value is obtained for this which is about ten times greater than for the basal plane; it is measured, of course, after slight basal glide. There is an order-of-magnitude preference for the basal plane, not only for glide but also for cleavage fracture.

Taking into account the critical normal stresses of basal and prismatic-plane type I it is now possible to calculate generally the
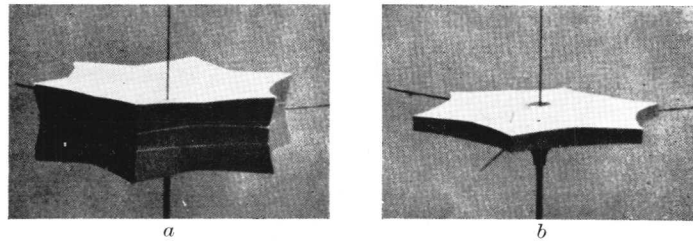

FIG. 141 (a) and (b).—(a) Tensile-strength Surface, and (b) Fracture Surface, of the Zn Crystal at $-185°$ C. (278).

dependence of the tensile strength upon orientation. The result is represented by the surface illustrated in Fig. 141a, which gives the tensile strength corresponding to various orientations and clearly reveals the strong influence of crystal orientation. A surface representing the tendency to fracture in a wider sense is shown in Fig. 141b. It connects the *initial* orientation of the crystal with its tensile strength with respect to brittle cleavage along the basal or prism planes, and (in far the greater part of the field of orientation) with its yield point with reference to extension by basal glide. This is given by the constant critical value of the shear stress in the glide system (Section 40).

For bismuth, too, the second-best cleavage plane—the rhombo-hedral plane $(11\bar{1})$—could be characterized quantitatively by means of its critical normal stress (285). It is true that, with the orientations investigated, mechanical twinning always started before fracture occurred, as could be detected from narrow striations on

occurs (obtained from experiments) does not usually apply for the undeformed initial state. It may depend very largely on the extent of previous deformation, as will be explained in greater detail in Section 54. At $-185°$ C., however, this dependence was found to be only slig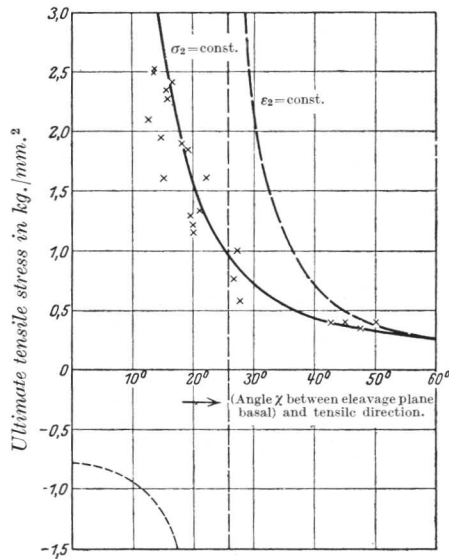ht, so that the tests at this temperature can be discussed here (278). Fig. 140 represents the tensile strength as a function of the angle of the cleavage plane. The observations again agree satisfactorily with the curve calculated on the assumption of the normal stress law. In particular, these tests with zinc reveal substantial disagreement between the results of the tests, and a fracture condition of constant *normal dilatation* perpendicular to the cleavage plane. A discontinuous line in Fig. 140 indicates the course of the tensile strength resulting from this assumption. Agreement between the two statements is assumed for the transverse position of the basal cleavage plane. It will be seen that constancy of the normal dilatation would demand a very much more pronounced dependence of the tensile strength upon orientation. For a position angle of $\sim 26° \left(tg\chi = \sqrt{-\dfrac{S_{13}}{S_{33}}} \right)$ an infinitely great tensile strength is obtained (dilatation resulting from longitudinal extension, and reduction of area resulting from transverse contraction of the test bar, are here equal to each other) and for a still more oblique position of the cleavage plane normal dilatation becomes possible only under compressive stress. Whereas in the case of the initial condition for

FIG. 140.—Tensile Strength of Zn Crystals at $-185°$ C. as a Function of Orientation [(278), (282)].

———: constant normal stresses : $\sigma = \dfrac{\epsilon_2}{\sin^2 \chi}$;

— — —: constant normal dilatation :

$$\sigma = \frac{\epsilon_2}{s_{11} \cos^2 \chi + s_{33} \sin^2 \chi}$$

The tensile strength increases, therefore, as the direction of tension approaches the hexagonal axis. Down to an angle of inclination of 29° for the most transverse (10$\bar{1}$0) plane the normal stress law continues to apply (Fig. 138). If the position of the cleavage plane becomes still more inclined, the strength of the material [1] is exceeded before the critical normal stress is reached. In this range of orienta-
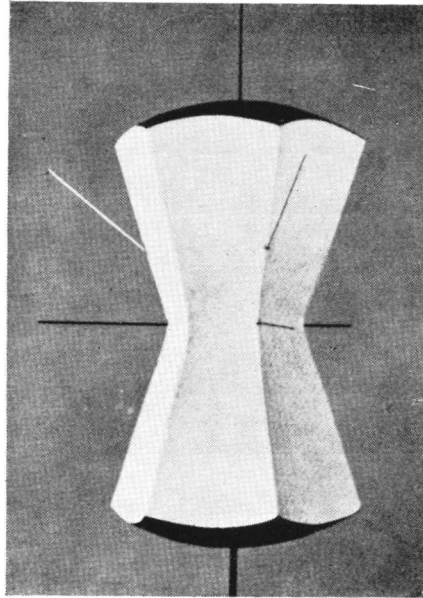

Fig. 139.—Tensile Strength Surface of the
Te Crystal (283).

tion the tensile strength remains noticeably constant at approx. 1·8 kg./mm.^{2} A spatial representation of the dependence of the tensile strength of tellurium crystals upon the orientation is given by the surface illustrated in Fig. 139.

Zinc crystals, which at 20° C. and at higher temperatures do not as a rule fracture along smooth crystal surfaces, exhibit at —80° C. (and at lower temperatures) smooth cleavage fractures along the basal plane. This fracture is mostly preceded by glide along the basal plane, which at —185° C. can amount to a glide of approx. 200 per cent. Therefore the basal normal stress at which fracture

[1] *Translator's footnote.* With respect to fracture processes other than cleavage along the (10$\bar{1}$0) plane.

which functions as cleavage plane (basal plane in the hexagonal representation). The curve calculated according to equation (53/1) on the assumption of a constant normal stress on the cleavage plane, agrees with the observations. Since the (111) plane is also the best glide plane of the bismuth crystal, the lattice positions which at 20° C. separate the area of brittle fracture from that of incipient
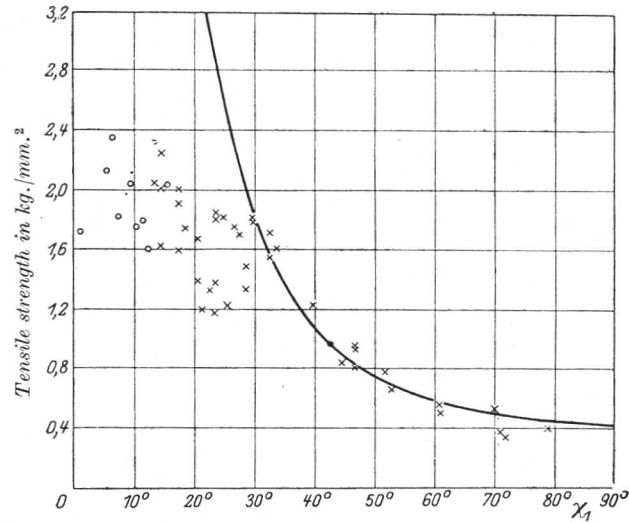

Fig. 138.—Tensile Strength of Te Crystals as a Function of Orientation (283). χ_1 angle between tensile direction and the most transverse prism plane type I.

glide can be calculated from the critical shear and normal stress of this plane. From the condition

$$\sigma = \frac{S}{\sin \chi \cdot \cos \lambda} = \frac{N}{\sin^2 \chi} \quad \text{the relation} \quad \frac{\sin \chi}{\cos \lambda} = \frac{N}{S}$$

is obtained for the boundary of the areas of glide and of fracture. With the aid of the critical stress values for bismuth we arrive at a limiting angle of 55° 40′, for $\chi = \lambda$. Where the initial position of the (111) plane is more transverse, it is the critical normal stress that will be reached first, while with more obliquely oriented crystals it will be the critical shear stress.

Results similar to those with bismuth were obtained with tellurium crystals, which exhibited tensile-strength differences of 1 : 4. The cleavage plane here is one of the (10$\bar{1}$0) planes (prism planes type I).

(see Fig. 136). This fracture either occurs without any previous perceptible deformation (brittle crystals), or it terminates a more or less large deformation the magnitude of which substantially affects the value of the tensile strength.

Nothing is yet known regarding the period of time covered by the process of rupture along cleavage planes. Although as a rule fracture occurs rapidly, it is, nevertheless, certain that the speed of the operation is finite (see Section 69). Markings on the cleavage planes and the occasional presence of flaws in the crystal fragments indicate that the process of fracture often consists in the gradual deepening of cracks.

53. Conditions Covering Brittle Fracture. Sohncke's Normal Stress Law

Tensile tests carried out on rock-salt crystals of various orientation led Sohncke to formulate the Normal Stress Law (277). According to this law these crystals break when a certain critical normal stress has been reached in the cubic-cleavage plane which is invariably the surface of fracture. The fracture strength therefore depends strongly upon the angle (χ) between cleavage plane and the direction of the tension. If N is the critical normal stress, equation (40/2) gives for the fracture strength σ the expression $\sigma = \dfrac{N}{\sin^2 \chi}$ (53/1). (For further particulars of experiments with rock salt see Chapter VII.)

FIG. 137.—Tensile Strength of Bi Crystals as a Function of Orientation (284).
$\times$ = at $-80°$ C., $\bigcirc$ = at $20°$ C.

Systematic experiments with metal crystals have shown that here, too, the occurrence of brittle fracture along smooth cleavage planes can readily be brought within the scope of the normal stress law. For example, *bismuth* crystals fracture in a brittle fashion at low temperatures and also at room temperature within a certain orientation range. Fig. 137 shows the extent to which the fracture strength of bismuth crystals depends upon the position of the (111) plane
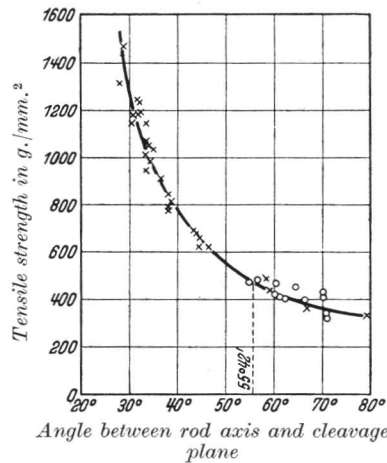
(a) α-Fe
(001).

(b) Zn
(0001).

(c) Bi
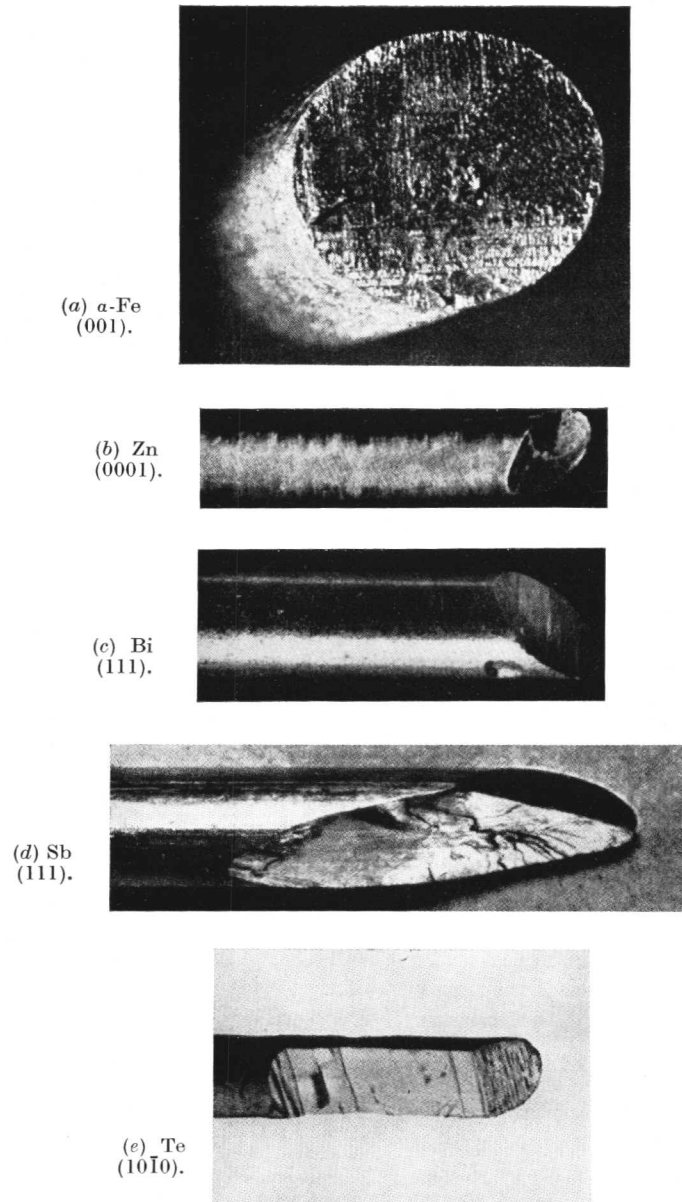(111).

(d) Sb
(111).

(e) Te
(10$\bar{1}$0).

Fig. 136 (a)–(e).—Fracture (Cleavage) Planes of Metal Crystals.

due to primary hardening by alloying, or to a substantial hardening of a latent glide system, in the present case it is the pronounced hardening of the basal plane due to twinning that is responsible for the reduction of its capacity for further hardening.

Scarcely any information is yet available on the question of *the influence of mechanical twinning on further twinning*—that is, whether a deformation twin or the initial crystal is more resistant to mechanical twinning. The answer depends on whether the plasticity which can be realized through repeated twinning is parallel or opposed to the direction of stress. It is also necessary to distinguish between the untwinning of a deformation twin and the development of a secondary twin which is distinct from the original crystal. Tests carried out on zinc sheets (274) show that it is possible, by bending backwards, to cause previously produced deformation twins to disappear; in this case, therefore, untwinning proceeds more easily than further twinning in the parent crystal. Similar observations have been made with antimony crystals (275). A more quantitative investigation of these relationships confirmed this result (276). In the course of an alternating bend test carried out on coarse-grained zinc sheets it was found possible to produce, and untwin, the same twin lamellæ as many as twenty times. Besides sudden broadening (or narrowing), in the course of these experiments a phenomenon was observed which, under the microscope, appeared to be a continuous change of the width of twin lamellæ with increasing (or again decreasing) bending moment. The moment required for the complete untwinning of a lamella is greater than that needed for its production. If the same lamella is repeatedly produced, weakening will be observed in so far as the required moment will diminish steadily. Consequently, if the test is repeated with equal bending moments, broader lamellæ will be produced. On the other hand, the bending moments required for untwinning appear to increase as the test is repeated.

This chapter shows that we have still a very great deal to learn about the dynamics of mechanical twinning.

D. FRACTURE ALONG CRYSTALLOGRAPHIC PLANES

Hitherto we have been concerned with crystal *deformation* processes, a consideration of which included glide and shear fractures. In the present section we shall discuss that type of rupture which consists of a fracture of the crystal along plane surfaces which coincide invariably with one of the cleavage planes listed in Table VI

it is transverse. Consequently, each of the three most obvious assumptions : *i.e.*, that either the true tensile stress, or the shear stress, or the normal stress in the twinning system, is responsible for the lower limit of the stress range in question, is compatible qualitatively with the experimental data relative to the width of the stress range that leads to twinning. The experiments show further, however, that the range within which twinning occurs decreases with increasing χ_0 angle much more rapidly than would follow from either of the three assumptions. If the effect is attributed to an increase of the lower stress limit, it is then possible to say that *twinning is made more difficult* by previous *glide*.
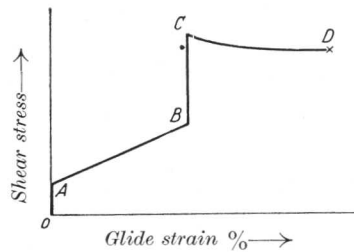

FIG. 135.—Sketch showing the Behaviour of the Resolved Shear Stress on the Basal Plane in the Course of Primary Glide (*AB*), Mechanical Twinning (*BC*) and Secondary Glide (*CD*) of Zn Crystals (272).

Only a few details are as yet available regarding the converse case of the influence of twinning on glide. An examination of zinc and cadmium crystals suggests that the shear stress of the basal plane experiences a considerable discontinuous increase as a result of twinning [(271), (272), (270), cf. also (273)]. This shear hardening by twinning amounts at room temperature to about 100 per cent. for zinc crystals, and about 200 per cent. for cadmium crystals. It is true that the resultant twin bands, especially in the case of cadmium, are usually very thin, so that it would appear possible that the mechanical properties are influenced by the size of the crystal investigated. Subsequent glide in the twin is accompanied by only a slight shear hardening on the basal plane, frequently even by a reduction of the shear stress (a softening during secondary basal glide occurred in nine out of twelve cases with zinc crystals, and in four out of twenty-three cases with cadmium crystals). A diagram showing the behaviour of the resolved shear stress of the basal plane of a zinc crystal is given in Fig. 135.

The fact that secondary glide on the basal plane is accompanied by only slight hardening (or even by some softening) furnishes a further example of the limitation of the hardening capacity of crystals. Analogous examples have already been noted in the glide of solid solutions (Section 46). Whereas in the case of solid solutions the small extent of the further increase of the yield stress was

maximum load if the position of the basal plane is rather transverse and the extension curve consequently flat.

This phenomenon can perhaps be explained as follows : if, taking as a basis the yield–stress curve which is valid for basal glide, we calculate the extension curves in the true tensile stress–strain diagram, the result shown for cadmium in Fig. 134 is obtained.[1] If allowance is made for the change in orientation which accom-
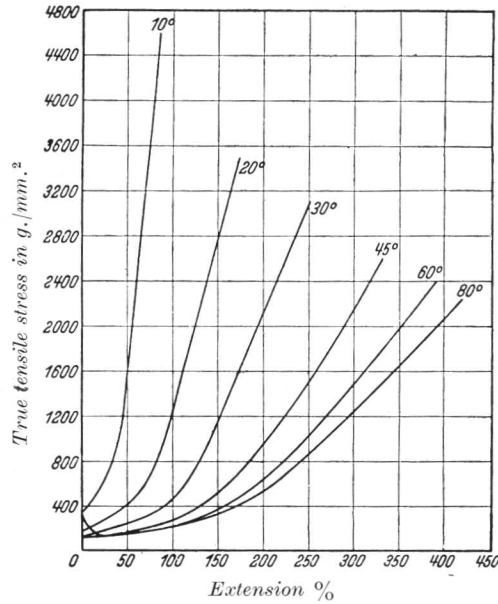

FIG. 134.—Calculated True Tensile Stress–Strain Curves for the Basal Glide of Cd Crystals of Various Orientations.

panies extension, it is also possible to deduce from these curves the behaviour of the shear and normal stress in the preferred twinning system. The curves obtained for these stresses exhibit in their general course no substantial difference from the true stress curves. In all three cases the high values are obtained with much smaller extensions if the initial position of the basal plane is oblique than if

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[1] The true stress $\sigma_{tr.}$ (referred to the current cross-section) is obtained from the " conventional " stress σ referred to the initial section by multiplying by $d = \dfrac{q_0}{q}$. In order to obtain the equation of the true stress curves, the equation (43/3) must be multiplied by d.

of the atoms from their starting points to their final position is also of importance. One fact does emerge, however, from the scanty material so far available regarding mechanical twinning of metal crystals, namely, that as with glide (see Section 42), the shear of the two neighbouring lattice planes is not essentially elastic. The elastic shear value which is reached at the onset of twinning is many orders of magnitude below that of the shear resulting from twinning. Let the cadmium crystal serve as an example : the reciprocal shear modulus in the twinning system is certainly not greater than 60×10^{-13} cm.^{2}/dyne, the maximum value for $1/G$. The shear stress needed to produce twin lamellæ in the twinning system is also substantially smaller than 500 g./mm.$^2 \cong 5 \times 10^7$ dyne/cm.^{2} (270), so that the elastic shear attained at the moment of twinning is
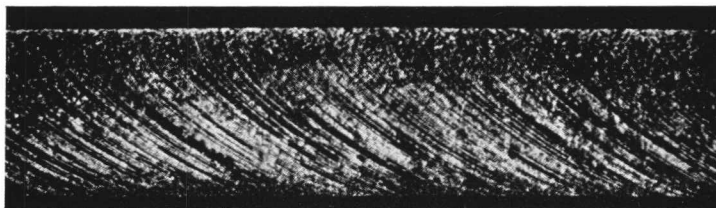

FIG. 133.—Fe Crystal Extended at $-185°$ C., Showing Deformation Twins. $K_1 = (112)$ (269a).

certainly smaller than 3×10^{-4} against $s_{Cd} = 0\cdot175$. Differences of the same order occur with other metals which form deformation twins.

Temperature appears to influence twinning even less than it affects the start of glide, for twinning is superseded by glide at high temperatures (Zn, Cd), but occurs preferentially at low temperatures (α-Fe ; Fig. 133).

52. *Mutual Influence of Twinning and Glide*

The width of the stress range in which mechanical twinning can occur in hexagonal crystals under static tensile stress depends to a very marked degree upon the orientation of the crystals, or rather upon the shape of the extension curve as determined by the orientation of the glide elements. This can be seen from Fig. 132, *b* and *c*, which shows that twinning starts early where the initial position of the basal plane is very oblique, and the extension curve therefore steep, but that it is restricted to a very narrow range just below the
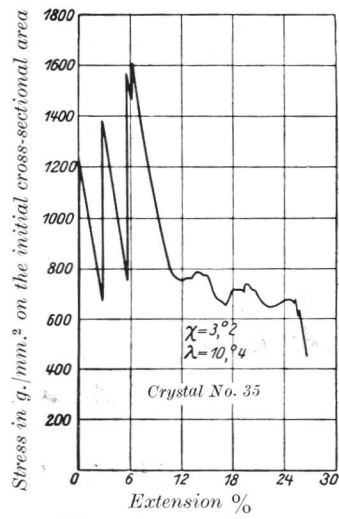
(a) Discontinuous change of load in the formation of deformation twins.
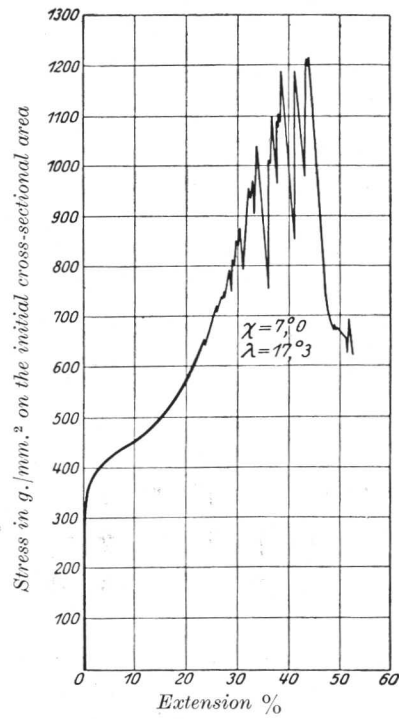
(b) Twinning after substantial glide.
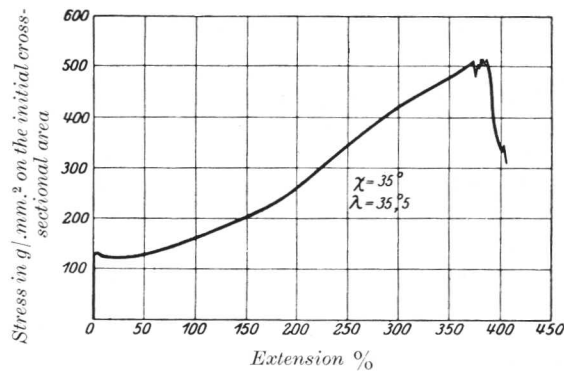
(c) Twinning after substantial glide.

FIG. 132 (a)–(c).—Stress–Strain Curves of Cd Crystals (270).