NATURAL SCIENCES TRIPOS Part IB
NATURAL SCIENCES TRIPOS Part II (General)

Monday 1 June 2009 09.00 to 12.00

MATERIALS SCIENCE AND METALLURGY (2)

Answer **five** *questions, not more than* **two** *being chosen from any one section.*

Write on **one** *side of the paper only, and begin your answer to each question at the top of a new sheet.*

Where questions are divided into a number of parts, the **approximate** *fraction of credit allocated to each part is indicated by the percentage in the right hand margin.*

Candidates using electronic calculators are advised to indicate clearly the sequence of steps in their working. Appropriate credit can then be given for the intermediate stages, even if the final stage is incorrect.

The answer to **each question** *must be tied up* **separately***, with its own cover–sheet.*

Write the relevant **question number** *in the square labelled 'Section'. Also, on* **each** *cover-sheet, list the numbers of* **all** *questions attempted.*

STATIONERY REQUIREMENTS
Metric graph–paper
Blue cover sheet × 5
A4 script paper
Rough work pad
Tags

SPECIAL REQUIREMENTS
Data Book (supplied by Department)
Approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION D

10.

(i) Explain why two samples fabricated from the same brittle material using an identical process can fail at different tensile loads. [15%]

(ii) Fifty samples were tested to failure in uniaxial tension by loading them in increments of 100 g. The accumulated mass at which each sample failed is listed below. Calculate the mean failure load. [10%]

Accumulated mass / g	Number of samples which failed
700	0
800	4
900	5
1000	7
1100	10
1200	11
1300	8
1400	4
1500	1

(iii) Make a Weibull plot of the data and calculate the Weibull modulus. [40%]

(iv) Estimate the load for which the probability of failure is a) 0.1 and b) 0.001. Comment on the relative confidence in these estimates. [15%]

(v) A new batch of samples is fabricated using the same processing method. They have the same cross sectional area, but are three times longer. Explain whether the average tensile strength is likely to increase, decrease or remain the same. Estimate the median tensile failure load of these longer samples. [20%]

11.

(i) Explain what is meant by the *principal stresses* of a stress state. Sketch the three Mohr's circles for a stress state with principal stresses σ_1, σ_2 and σ_3, and indicate how these can be used to establish the maximum shear stress and the orientation of the plane(s) on which it acts. [25%]

(ii) An aluminium alloy drive shaft of an aircraft is designed as a solid cylinder of 20 mm diameter. It is expected to be subjected to a torque of 100 N m and an axial tensile force of 10 kN. Using a Mohr's circle, or otherwise, evaluate the two principal stresses acting in the surface of the shaft, and the angle between the larger of these and the axis of the shaft. [50%]

(iii) Stating any assumptions, calculate the minimum uniaxial yield stress required of the alloy if plastic deformation is to be avoided. [25%]

[The shear stress at the free surface of a solid cylinder, of radius R, generated by the application of a torque, T, is given by $\sigma_{\theta z} = RT/I_p$, where I_p is the polar second moment of area given by $\pi R^4/2$.]

12.

(i) Explain what is meant by a bending moment, M, and show that the moment acting in the section of a beam bent to a curvature κ is given by $M = \kappa EI$, where E is the Young's modulus of the material and I is the second moment of inertia of the section of the beam. [25%]

(ii) Show that the second moment of area of a rectangular section beam, of width w and thickness h, is given by $I = wh^3/12$. [15%]

(iii) Derive an expression for the deflection of an end-loaded cantilever beam, as a function of distance along it. Hence show that the vertical displacement half way along the beam is 5/16 of the deflection at the end. [30%]

(iv) The component of an atomic force microscope which contacts the specimen is a cantilever made of silicon ($E = 150\,\text{GPa}$), with a width of 50 μm, a thickness of 5 μm and a length of 300 μm. The maximum deflection of the end of the cantilever is 20 μm. Calculate the force that must be applied to the end of the cantilever in order to generate this height change. [30%]

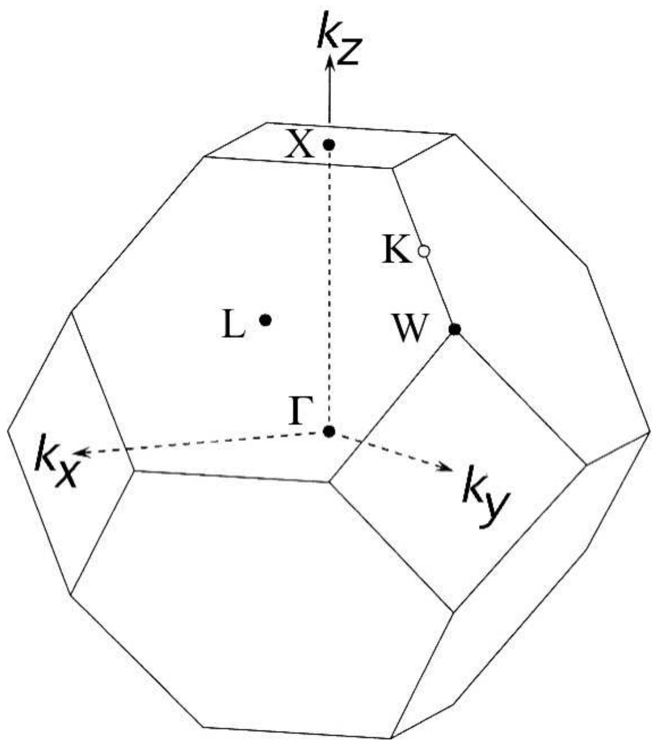
SECTION E

13.

(i) The first Brillouin zone for the cubic close–packed crystal structure is shown below. How many faces does the shape have, and what are their {hkl} indices? Indicate the number of mirror planes, four–fold, three–fold and two–fold rotation axes. [20%]

(ii) Label the wavevectors of the positions L, X, W and K relative to the coordinates k_x, k_y and k_z. [35%]

(iii) Show that the distances Γ–L, L–K, K–W and W–Γ have magnitudes in the ratios $2\sqrt{3} : \sqrt{6} : \sqrt{2} : 2\sqrt{5}$. Sketch and account for the E–k relationship for free–electron wavevectors in the first Brillouin zone for the circuit Γ–L–K–W–Γ, plotting the circuit as a single horizontal axis, so that the magnitudes of the distances involved are to scale, with $E(k)$ as ordinate. [45%]



14.

(i) In terms of flat band diagrams, differentiate between *simple metals*, *intrinsic semiconductors, extrinsic semiconductors* (n–type and p–type) and *insulators.* Indicate the Fermi level in each case. [30%]

(ii) The Fermi level of an extrinsic semiconductor is coincident with the donor levels at 240 K. Raising the temperature to 300 K changes the occupation probability of the donor levels by 0.3. Determine where the Fermi level lies relative to the donor level at 300 K. [30%]

(iii) The electron energy levels in a particular semiconductor are free–electron–like with respect to the bottom of the band at $E = 0$, so that for $E > 0$, the number of occupied states takes the form:

$$n(E) = \frac{1}{2\pi^2}\left(\frac{2m}{\hbar}\right)^{\frac{3}{2}} E^{\frac{1}{2}} \exp\left(-\frac{E - E_F}{k_B T}\right)$$

Determine the most highly populated electron energy in the conduction band relative to $E = 0$. Find the average electron energy. [40%]

15.

(i) Explain what is meant by a *reciprocal lattice* and interpret the physical meaning of a reciprocal lattice vector. Express the reciprocal lattice vectors $\mathbf{a}^*$, $\mathbf{b}^*$ and $\mathbf{c}^*$ in terms of the real space vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$. [15%]

(ii) Show that when a direction with real space indices $[uvw]$ lies in a plane (hkl),

$$hu + kv + lw = 0$$

irrespective of the symmetry of the lattice. [20%]

(iii) What are the angles in a hexagonal lattice between: $\mathbf{a}^*$ and $\mathbf{b}^*$; $\mathbf{a}^*$ and $\mathbf{c}^*$; and $\mathbf{b}^*$ and $\mathbf{c}^*$? [15%]

(iv) What are the angles between: $\mathbf{a}^*$ and $\mathbf{b}^*$; $\mathbf{a}^*$ and $\mathbf{c}^*$; and $\mathbf{b}^*$ and $\mathbf{c}^*$; in a monoclinic lattice in which the obtuse angle is β? [15%]

(v) Determine the spacings of the (002), (110) and (012) planes in a monoclinic crystal with $a = 2.885$ Å, $b = 4.120$ Å, $c = 4.622$ Å and $\beta = 96.8°$. [35%]

SECTION F

16.
An alloy of composition 95Al–5Cu wt% is solidified in a sand mould. Representative microstructures are shown below.

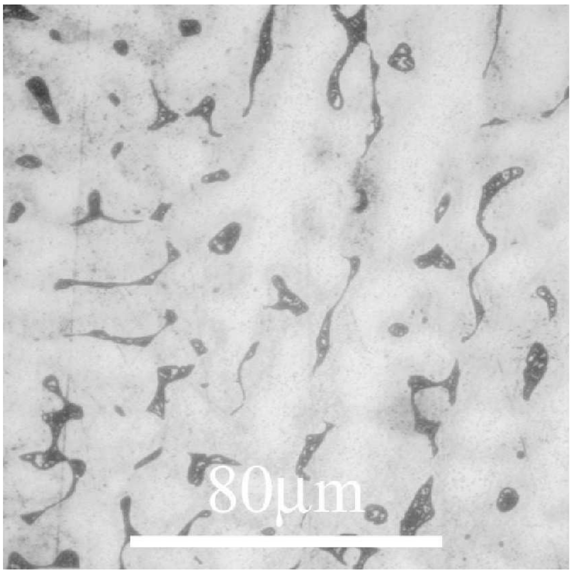
(i) Sketch the important features of the microstructures illustrated, indicating the phases present. [25%]

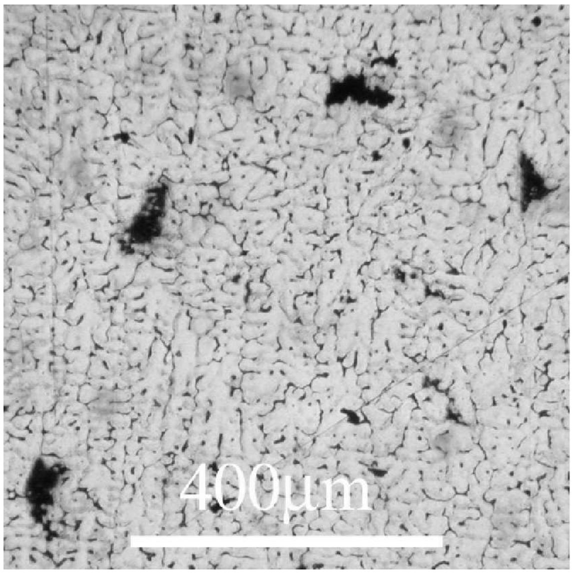
(ii) Explain the development of the microstructure during cooling from the liquid to room temperature. [25%]

(iii) Calculate the fraction of the alloy expected to have the eutectic composition if the casting conditions permit perfect mixing in the liquid but allow negligible diffusion in the solid. Explain the relevance of the result to the micrographs shown below. [25%]

(iv) Explain the annealing treatment needed to homogenise the microstructures illustrated below. [25%]

[Inter-diffusivity of Al and Cu: $D = 5\times10^{-6}\exp(-Q/RT)\,\mathrm{m^2\,s^{-1}}$, $Q = 120\,\mathrm{kJ\,mol^{-1}}$]





17. Answer **ALL** of the following questions. [All questions carry equal marks]

(a) Using energy level diagrams, show how an Ohmic contact can be formed between a metal and an n–type semiconductor.

(b) Why is the separation of the anodic and the cathodic reactions in the aqueous corrosion of steel larger than during its oxidation?

(c) Explain why an interfacial polycondensation process would not be a good method for producing high strength nylon fibres.

(d) Explain the basis of energy dispersive X–ray analysis as implemented in a scanning electron microscope.

18. Answer **ALL** of the following questions. [All questions carry equal marks]

(a) Explain the appearance of polymer spherulites under polarised light, in particular the Maltese Cross pattern and banding that can occur, in terms of the molecular structure of the polymer chains.

(b) Explain the physical basis of infrared spectroscopy, commenting on the types of materials for which it is most suited.

(c) Stating any assumptions, show that during elastic deformation the fractional change in volume is equal to the sum of the linear strains in three perpendicular directions. Hence prove that volume is conserved during elastic uniaxial–tension only when the Poisson's ratio is 0.5.

(d) It is proposed to build a house on top of the trunk of a spruce tree which is 4 m high and 20 cm in diameter. Calculate the maximum load which could in theory be borne by the tree. Comment on your answer and explain why the real failure load is likely to be different.

END OF PAPER